THERMO-MECHANICAL RESPONSES OF METALS ON THE FAST-TRANSIENT PROCESS IN SMALL VOLUMES

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Summary This work aims at developing a multi-scale thermodynamically consistent framework to address the mechanical and thermal behavior of metallic components in small scale and fast transient time. This model is capable of describing the size effects exhibited by metallic components subjected to inhomogeneous plastic flow, when diminishing size from a few tens of micrometers to a few hundreds of nanometers, with smaller being stronger. Moreover, the effect of short pulse of heating on the energy transport and thermal responses of the metals is incorporated in the framework.

INTRODUCTION

The modelling of plastic deformations in materials become more complex when it is subjected to inhomogeneous plastic flow and its size reduces from a few tens of micrometers to a few hundreds of nanometers where the microstructure characteristic length is greater than the required resolution length or where the size of the representative volume element (RVE) is significant compared with the specimen size. As traditional continuum mechanics do not contain characteristic lengths, the use of nonlocal models (i.e. strain gradient plasticity) is required in order to introduce a microstructure characteristic length (long-range microstructural interactions) where the stress response at a material point is assumed to depend on the state of its neighbourhood in addition to the state of the material point itself. On the other hand another issue that needs to be taken into account is the effect of the higher order temperature gradient on the diffusion of atoms and heat in the fast-transient behavior. This effect becomes significant when the rate of deformation is high enough not to allow heat to dissipate. The validity of the classical heat equation can be questioned in such situations since the effect of phonon-electron interactions in this time frame, needs to be incorporated in the heat transfer model. This gives rise to a large variety of generalized (microscopic) heat transport equations. In this work the aforementioned phenomena is coupled to address the thermo-mechanical behavior of metallic components in small scale and fast transient time.

THERMODYNAMICALLY CONSISTENT FRAMEWORK

Thermo-mechanical behaviour of metals including the effect of strain and temperature gradient is addressed through a thermodynamic consistent formulation. The principle of virtual power ($P_{ext} = P_{int}$) is used to derive the governing micro-force balance equation which when augmented by the constitutive relations results in the yield criterion or the plasticity loading surface. The external power is balanced by an internal expenditure of power characterized by the Cauchy stress tensor σ_{ij} defined over V, the drag stress R conjugate to the accumulated plastic strain rate $\dot{p}$ and the micromorphic generalized stress $\mathcal{A}$ conjugate to $\dot{T}$. However, one can assume that additional powers are expended internally by the higher-order microforce vector Q_k and micromorphic generalized stress vector $\mathcal{B}_i$ conjugate to the rate of the gradient of stress $\nabla\dot{p}$ and temperature $\nabla\dot{T}$.

$$P_{int} = \int_V (\sigma_{ij}\dot{\varepsilon}^{e}_{ij} + R\dot{p} + Q_k\nabla\dot{p} + \mathcal{A}\dot{T} + \mathcal{B}_i\nabla\dot{T})\mathrm{d}V \tag{1}$$

In order to address the effect of the multiple sources of thermodynamic processes into modelling, the mechanical state variables are decomposed into energetic and dissipative components with the corresponding dual parameters for energetic and dissipative gradient length scales. By introducing the Helmotz free energy such that $\psi = \psi(\varepsilon^{e}_{ij}, p, \nabla\dot{p}, \dot{T}, \nabla\dot{T})$ and substituting into the *entropy production inequalities* the energetic part of the thermodynamic forces can be obtained. However, the yield function is treated here as the dissipation potential providing the flow and evolution rules for internal variables. Substituting the obtained thermodynamic forces into micro-force balance equation, results in the following expressions for the nonlocal form of the yield criterion:

$$\tau = R + \sigma_Y\frac{\dot{p}}{\dot{\mathcal{P}}} - G\ell_{en}^2\nabla^2 p - \ell_{dis}^2\frac{\partial}{\partial x}[\sigma_Y\frac{\dot{p}}{\dot{\mathcal{P}}}] \tag{2}$$

where ℓ_{en} and ℓ_{dis} are energetic and dissipative length scales respectively and the effective nonlocal flow rate is defined as $\dot{\mathcal{P}} = \sqrt{\dot{p}^2 + \ell_{dis}^2\nabla\dot{p}\nabla\dot{p}}$.The crystal structure effect is included in the formulation by different functions for σ_Y and R. Bcc metals show stronger dependence of the yield stress on temperature and strain rate while in the case of fcc metals the yield stress is mainly affected by strain hardening [1]. Metal alloys response is the combination of the both aforementioned behavior. Therefore by additionally decomposed flow stress into equivalent athermal stress Y_a and equivalent thermal stress Y_{th}, for metal alloys one obtains:

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$$\sigma_Y = Y_a + \hat{Y}\mathcal{G}(T, \dot{\mathcal{P}}) \;\; and \;\; R = R_a(\dot{\mathcal{P}}) + \hat{R}\mathcal{G}(T, \dot{\mathcal{P}}) \tag{3}$$

The temperature and rate dependent function $\mathcal{G}(T, \dot{\mathcal{P}})$ vanishes in σ_Y and R functions for fcc and bcc metals respectively. Moreover, using the first law of thermodynamic and considering the gradient of temperature the following generalized heat equation can be obtained which accounts for the effect of microscale in space and in time as follows [2]:

$$\dot{T} = \frac{\chi}{\rho c_p}(R^{dis}\dot{p} + Q_k^{dis}\dot{p}_{,k}) + (\alpha\nabla^2 T + \alpha\tau_T\nabla^2\dot{T} - \tau_q\ddot{T}) \tag{4}$$

where τ_T and τ_q are two positive intrinsic time scales characterizing the effect of the microstructural interactions and the fast transient response respectively.

NUMERICAL EXAMPLE FOR ONE-DIMENSIONAL PROBLEM

The proposed theory is implemented here for a thin film on thick substrates under uniaxial straining to investigate the appropriate length scale and time scale effects on the mechanical and thermal behavior of small structures. Boundary layer formation of plastic strain through the film thickness is shown in Figure 1 for different length scales ratio (e.g. divided by the film thickness). Moreover, it is clearly seen in Figure 1(c) and (d) that the energetic length scale gives rise to the strain hardening while the dissipative length scale increases the yield stress. However, due to addition of the temperature in the formulation and in higher amount of dissipative length scale, thermal softening occurs in the materials.

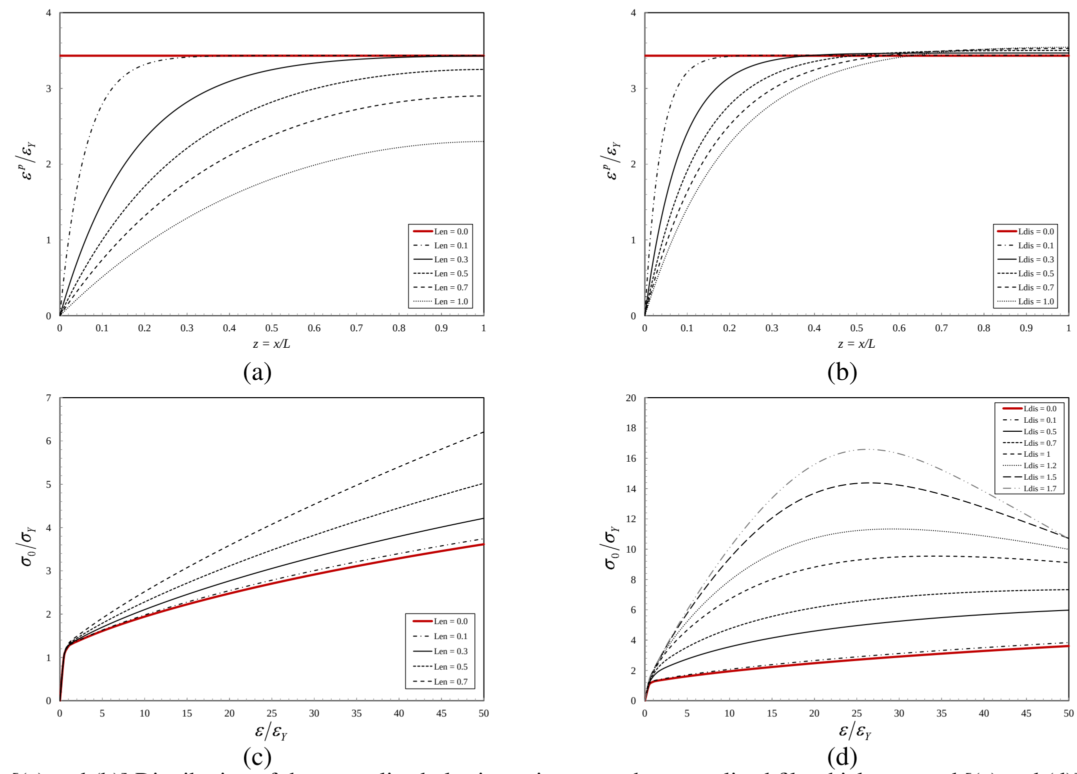

Figure 1. [(a) and (b)] Distribution of the normalized plastic strain across the normalized film thickness, and [(c) and (d)] normalized stress versus the applied strain. Size effects due to the bulk length scale are described for the case of [(a) and (c)] for different values of L_{en} while $L_{dis} = 0.0$ and [(b) and (d)] for different values of L_{dis} while $L_{en} = 0.0$. In all the curves a constant temperature at interface ($T^I\mid_{x=0} = T_0 = 77K$) is assumed.

CONCLUSION

Introduction of various length and time scales is important to satisfactorily explain the size and rate dependency, hardening and strengthening effects and the boundary layer in metals.

References

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[2] Voyiadjis, G.Z. and D. Faghihi *Int J of Plasticity* , 2011.