EFFECT OF STRESS-STATE AND SPACING ON VOIDS IN A SHEAR-FIELD

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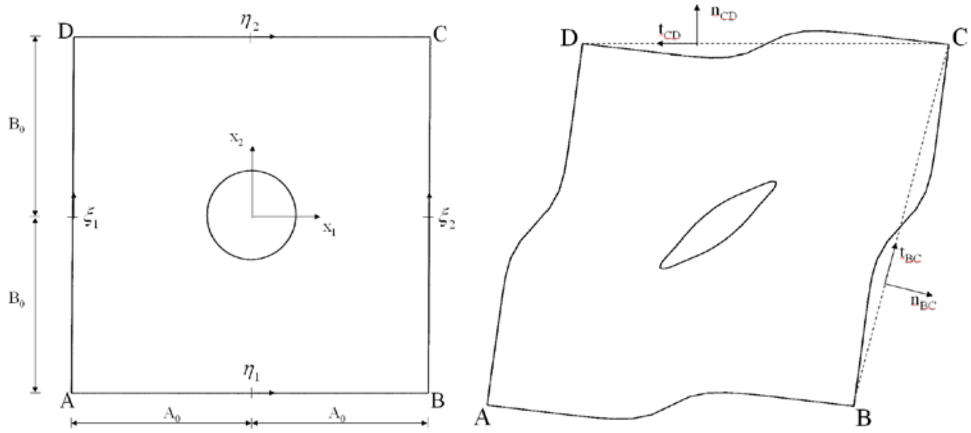
Summary Unit cell model analyses are carried out for a material with a periodic array of voids, subject to shear loading. The focus is on ductile fracture at low stress triaxiality or even negative stress triaxiality. The voids in shear flatten out to micro-cracks, which rotate and elongate until interaction with neighboring micro-cracks gives coalescence. The unit cell has fully periodic boundary conditions, allowing for any combination of the stress components and different void spacing in two directions.

INTRODUCTION

When voids deform in a ductile material subject to hydrostatic tension the void volumes tend to grow large and ductile failure occurs by coalescence of neighboring voids. However, in simple shear where the hydrostatic tension is zero, none of the standard models predict void growth or ductile failure. For materials undergoing shear deformations with low or even negative hydrostatic tension the behavior is very different. Thus, Fleck and Hutchinson [1] have found for an initially spherical void in a linearly viscous material under remote shearing that the void becomes ellipsoidal, rotates, and finally forms a penny-shaped crack. Recent plane strain micro-mechanical studies by Tvergaard [2,3] for a material containing a row of circular cylindrical voids subject to simple shear have shown that the voids are flattened out to micro-cracks, which rotate and elongate until interaction with neighboring micro-cracks gives coalescence. Thus, also in shear ductile failure occurs due to the deformation of voids and their interaction with neighboring voids, but the mechanism is very different from the well-known void growth to coalescence mechanism under tensile loading.
In [3] the void closure was accounted for in an approximate manner, by using an internal load on the void surface to simulate the crack surface contact, with the load level continuously adjusted such that the aspect ratio of the flattened void does not pass a specified limiting value. The approximation was made to avoid numerical complications in a full analysis of a completely closed crack with crack surface contact and with much material flow around the crack tips. Very recent studies [4], using a commercial code with full numerical contact modeling, have confirmed that the procedure in [3] gives a very good approximation to the condition of contact with frictionless sliding.
In the present paper the effect of the stress state and of the unit cell aspect ratio are studied for voids in a shear field. When the voids collapse to a micro-crack, the approximate description of [3] for contact with frictionless sliding is still applied, but the interaction with neighboring voids is not limited to a single row of voids, and the studies are not limited to cases with zero overall straining along the row of voids.

PROBLEM FORMULATION

The materials studied here have a periodic array of voids, with the initial spacing $2A_0$ in the x^1-direction and the initial spacing $2B_0$ in the x^2-direction. This periodic array is analyzed by considering a unit cell. Plane strain

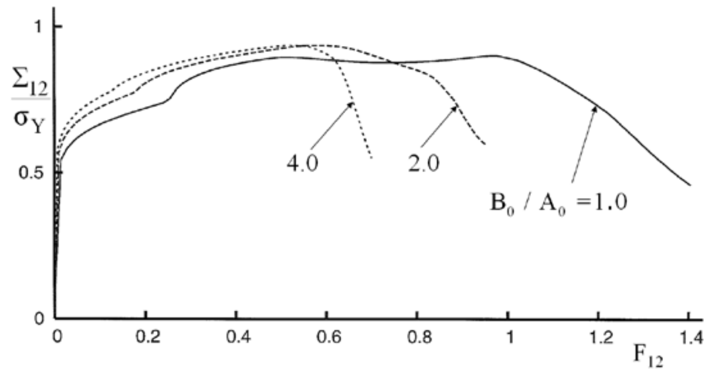


conditions are assumed and the voids are initially circular cylindrical with radius R_0. Finite strains are accounted for and the analyses are based on a convected coordinate Lagrangian formulation of the field equations, with a Cartesian x^i coordinate system used as reference. A finite strain formulation for a J_2 flow theory material with the Mises yield surface is applied. The true stress-logarithmic strain curve in uniaxial tension is taken to follow a power

law with Young's modulus E, the initial yield stress σ_Y and the power hardening exponent N. Periodic boundary conditions are prescribed for the unit cell.
On a deformed unit cell the average directions of the side BC and the top CD are indicated by dashed lines in the Figure, and the average normal and tangential vectors are indicated. By integration of the nominal tractions the resultant forces in the coordinate directions are determined for BC and CD, and using the current dimensions of the unit cell the average true stresses in the normal and tangential directions for BC and CD are determined. Finally, using standard relations for these normal and tangential stress components, the Cartesian true stress components Σ_{11}, Σ_{22} and Σ_{12} are calculated, and fixed stress ratios are prescribed $\Sigma_{22} / \Sigma_{12} = \kappa$, $\Sigma_{11} / \Sigma_{12} = \gamma$.
When the void deforms during shearing with low stress triaxiality, the length ℓ of the ellipsoidal cross-section of the void is calculated as the largest distance between two surface points, and the average width w of the void is calculated from the current void volume. Instead of a detailed representation of crack surface contact as the void develops into a micro-crack, the approximate procedure of [3] is applied to enforce that the average aspect ratio of the void satisfies the inequality $w / \ell \geq \rho$.

RESULTS

The elastic-plastic material analyzed has $\sigma_Y / E = 0.002$, and $N = 0.1$. Different values are considered for the initial aspect ratio of the region analyzed, the initial void radius to width ratio, the stress ratios κ and γ, and the limiting void aspect ratio ρ. For each case the average shear stress Σ_{12} is plotted against the deformation gradient F_{12}.



As an example results are shown here for $R_0 / A_0 = 0.25$, $\rho = 0.15$, $\kappa = 0$ and $\gamma = 0.1$. The different values of the aspect ratio B_0 / A_0 represent different void spacing in the vertical and horizontal directions. For the spacing 1.0 there is strong interaction with neighboring voids in both directions, as seen by the strong distortion of all edges of the deformed unit cell above, while for the largest spacing 4.0 there is practically no interaction with the neighboring voids in vertical direction. Strong sensitivity to the difference in void spacing is illustrated by this figure. The most important result in all the cases is that a maximum shear stress is reached, so that ductile fracture will occur by plastic flow localization. The mechanism in all cases is that voids collapse to micro-cracks, which rotate and elongate in the shear field until the onset of coalescence with neighboring micro-cracks. The early kink on the curves at a deformation gradient around 0.2 marks the point where crack surface contact initiates.
Previous studies [2,3,4] were limited to zero straining in the horizontal direction, and clamped edges at top and bottom, while here the value of the stress component in horizontal direction is specified instead. Several analyses here have been carried out for $\kappa = 0$ and $\gamma = 0.1$, *i.e.* for a stress state close to simple shear, but also larger tensile or compressive stresses in the horizontal or vertical directions have been considered. Generally, an increased stress triaxiality promotes earlier failure, while a negative stress triaxiality tends to delay failure. But the effect of these stress variations is found to be smaller for the low aspect ratio unit cell than for higher aspect ratios, which represent a higher initial void spacing in the vertical direction than in the horizontal direction.

References

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