THE EFFECTS OF RESIDUAL SURFACE STRESS ON THE ELASTIC FIELDS OF HALF-PLANE PROBLEMS

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Summary Based on Fourier integral transforms techniques, this short paper investigates the elastic stress fields of semi-plane problems with surface stress effects. Surface constitutive relations and generalized Young-Laplace equations, expressed in the reference configuration, are used to describe the elastic behaviour of the material surface. Due to the existence of residual surface stress, the Lagrangian expressions of surface stresses will have out-plane terms. The solutions with out-plane terms and without out-plane terms are analyzed. Numerical results show that in consequence of residual surface stress, the out-plane terms of surface stress have significant effects on the shear stress of a vertical concentrated loading case, but have little effects on the shear stress of a horizontal concentrated loading case.

INTRODUCTION

With the developments of science and technology, nano-scale structures and devices are widely used in nano electromechanical systems (NEMS). Due to the large surface-to-volume ratio characteristic, surface effects play an important role on the mechanical properties of these structures. Recently, many researchers study the effects of surface/interface stresses on elastic fields near the surface of half-plane and half-space [1-4]. These results show that the surface stress effects have significant influences on the elastic field near the surface of nano-scale structures.
When the surface constitutive relations are expressed in the reference configuration, the surface stresses will have out-plane terms due to the existence of residual surface stress [5,6]. This paper investigates the semi-plane problems loaded by a vertical and a horizontal concentrated force at the surface, and elucidate the effects of out-plane terms of surface stress.

GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

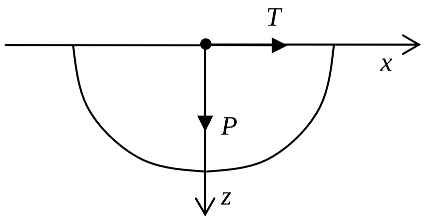

Fig.1. Semi-plane loaded by a vertical point force and a horizontal point force at the surface

General equations

Semi-plane problem loaded by a vertical and a horizontal concentrated force is shown in Fig.1. The generalized Young-Laplace equations and the constitutive relations for the material surface are expressed as [5, 6]

$$\sigma^s_{i\alpha,\alpha} + \sigma_{i3} = 0, \tag{1}$$

$$\sigma^s_{\alpha\beta} = \gamma^*_0\delta_{\alpha\beta} + \left(\gamma^*_0 + \gamma^*_1\right)\varepsilon^s_{\kappa\kappa}\delta_{\alpha\beta} - \gamma^*_0 u_{\beta,\alpha} + \gamma_1\varepsilon^s_{\alpha\beta}, \qquad \sigma^s_{3\alpha} = \gamma^*_0 u_{3,\alpha}, \tag{2}$$

where $\sigma^s_{i\alpha}$ is the surface stress, σ_{i3} denotes the bulk stress components, $\delta_{\alpha\beta}$ is identity tensors on the tangent planes of the surface before deformations; $\varepsilon^s_{\alpha\beta}$ denotes the surface strain components, $u_{\beta,\alpha}$ and $u_{3,\alpha}$ denote the surface displacement gradients of in-plane and out-plane terms, respectively; γ^*_0 is residual surface stress, γ^*_1 and γ_1 are surface elastic constants; the indices α take the value of 1 or 2, i take the value from 1 to 3.
The basic equations for the bulk material are

$$\sigma_{ij,i} + F_j = 0, \tag{3}$$

$$\sigma_{ij} = \lambda\varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij}, \tag{4}$$

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \tag{5}$$

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where σ_{ij} is the bulk stress components, F_i represents the body forces and in this paper we assume that $F_i = 0$, ε_{ij} denotes the strain components, u_i is the displacement components, λ and μ denote the bulk Lamé constants, δ_{ij} is the Kronecher delta symbol. It should be pointed that we neglect the residual stress in the bulk here [5, 6].

Boundary conditions

At $z = 0$, the boundary conditions can be written as

$$\sigma_{zz} + \sigma^s_{xz,x} + P\delta(x,z) = 0, \tag{6}$$

$$\sigma_{xz} + \sigma^s_{xx,x} + T\delta(x,z) = 0. \tag{7}$$

Based on these boundary conditions and general equations, the expressions of stresses and displacements can be obtained through Fourier integral transforms.

NUMERICAL RESULTS AND DISCUSSIONS

For numerical calculations and comparisons, we introduce an intrinsic length L, which is a surface characteristic parameter, and take the Lamé parameter for the semi-plane material as $\lambda = 2.226\mu$ (e.g. Aluminum[7]). Fig.2 shows the distributions of dimensionless shear stresses at $z/L = 0.1$ under the action of a vertical concentrated loading. The symbol "out-plane" denotes the solutions considered the out-plane terms, "no out-plane" represents the solutions without the out-plane terms and "classical" corresponds to the classical solutions without surface stress effects. It can be found that both the surface energy and out-plane terms have significant effects on shear stress near the surface. Fig.3 shows the distributions of dimensionless shear stresses at $z/L = 0.1$ under the action of a horizontal point force. It is shown that the surface energy have important effects on the shear stress but the out-plane term almost has no impacts on the shear stress.

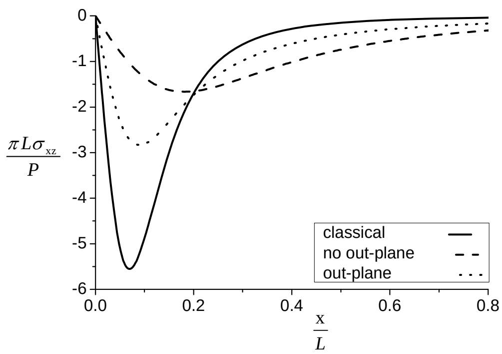

Fig.2. Shear stress contribution at $z/L = 0.1$ under vertical load

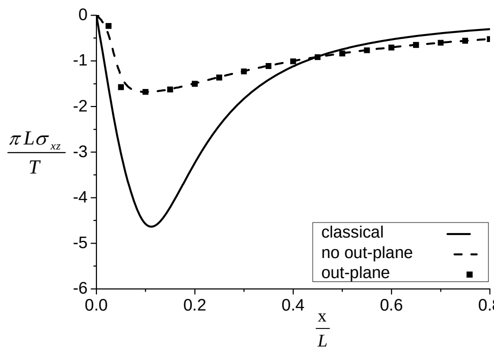

Fig.3. Shear stress contribution at $z/L = 0.1$ under horizontal load

Results show that the surface stresses play an important role in the elastic fields near the surface of semi-plane problems. Due to the existence of residual surface stress, out-plane terms have significant effects on half plane when loaded by a vertical point force, but little effects when loaded by a horizontal point force.

ACKNOWLEDGEMENTS

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