SIZE-DEPDENDENT MODELS FOR ELECTROSTATICALLY ACTUATED MEMS

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Summary Size-dependent microbeam model and microplate model are proposed for electrostatically actuated MEMS using strain gradient elasticity theory. Results show that the normalized pull-in voltage predicted by the new models increases nonlinearly with the decrease of the structure characteristic size, exhibiting size effect; while the corresponding classical models do not exhibit such a size effect. Very good agreement is found between the present model and available experimental data.

INTRODUCTION

The higher precise characterization of mechanical and electric properties for electrostatically actuated MEMS (Micro-Electro-Mechanical-System) has recently received considerable attention. Typical electrostatically actuated MEMS is comprised of a conductive deformable electrode suspended above a fixed electrode. Under an applied voltage, the deformable electrode deflects due to electrostatic attraction. When the voltage increases beyond a critical value, the movable electrode becomes unstable and collapses onto the fixed electrode, which is known as pull-in instability.

Much works have been conducted to establish models to characterize the mechanical and electrical properties using the classical beam theory [1] and classical plate theory [2]. However, as the size of MEMS continues to decrease, the classical elasticity continuum theory may be inefficient to describe their mechanical and electric behaviours, because size effect has been experimentally found for various materials.

Strain gradient elasticity (SGE) theory proposed by Lam et al. [3] is capable of capturing such size effect. The paper aims to develop size-dependent models for mcirobeam and microplate-based MEMS based on SGE theory.

SIZE-DEPDENENT MICROBEAM MODEL

According to Lam's SGE theory, the strain (ε_{ij}), strain gradients (γ_i, $\eta_{ijk}^{(1)}$, χ_{ij}^{s}) and the corresponding stress (σ_{ij}), higher stresses (p_i, $\tau_{ijk}^{(1)}$, m_{ij}^{s}) can be derived by using the displacement field of Bernoulli–Euler beam. Then strain energy U for SGE theory can be obtained via the following equation

$$U=\frac{1}{2}\int_V \bar{u}\,d\Psi=\frac{1}{2}\iiint_V(\sigma_{ij}\varepsilon_{ij}+p_i\gamma_i+\tau_{ijk}^{(1)}\eta_{ijk}^{(1)}+m_{ij}^{s}\chi_{ij}^{s})dxdydz \tag{1}$$

The electrical energy U_e between two electrodes is given by

$$U_e=\frac{1}{2}\int_l\frac{\bar{\varepsilon}bV^2}{(d-w)}dx \tag{2}$$

where w is transverse deflection, d the gap between two electrodes, V applied Voltage, $\bar{\varepsilon}$ the dielectric constant of the gap medium.

With the help of principle of minimum potential energy, the governing equation of static deformation for electrostatically actuated microbeam is obtained as

$$-\mu I(2l_0^2+\frac{4}{5}l_1^2)\frac{\partial^6 w}{\partial x^6}+(EI+\mu A(2l_0^2+\frac{8}{15}l_1^2+l_2^2))\frac{\partial^4 w}{\partial x^4}=\hat{N}\frac{\partial^2 w}{\partial x^2}+\frac{\varepsilon bV^2}{2(d-w)^2} \tag{3}$$

whered $\hat{N}$ is axial load, A and I are the area and moment of inertia of the cross section, E is Young's modulus, μ shear modulus, l_0, l_1, l_2 are material length scale parameters (MLSPs).

For the fixed-fixed microbeam, the corresponding boundary conditions are (l is length of beam):

$$w(0)=0\quad w(l)=0\quad \frac{\partial w(0)}{\partial x}=0\quad \frac{\partial w(l)}{\partial x}=0\quad \frac{\partial^2 w(0)}{\partial x^2}=0\quad \frac{\partial^2 w(l)}{\partial x^2}=0 \tag{4}$$

When size effect is ignored, Eqs. (3) and (4) will reduces to ones of classical beam model[1].

In order to evaluate the validity of the present analytical model, we compare the results of the present model with the experimental data measured in Ref. [4] as well as that predicted by the classical model [5] in Figure 1. The parameters are from Ref. [4]. It is seen that the classical model predicts a pull-in voltage of 27.28V, which is smaller than the experimental data 27.95V reported in Ref. [4], which may not be sufficient for accurate MEMS performance or control. In the present model incorporating the size effect, when the material length scale parameter Cl=0.11μm (here, without losing generality and for convenience, we assume $l_0= l_1= l_2=Cl$), the resulting pull-in voltage is 28.02V, which agrees well with the experimental data. Very good agreement is found between the present model and available experimental data. Thus, the new beam model can prove robust for describing the behaviour of size-dependent pull-in instability for microbeam-based MEMS devices.

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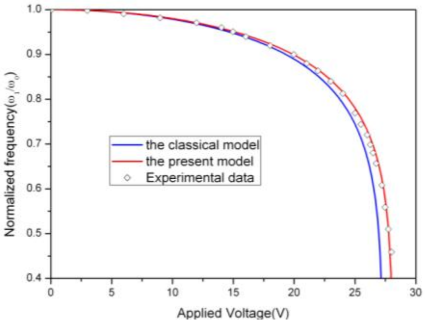

Figure 1. Comparison of the first-order normalized frequency between results of the present model, the classical model and experimental data.

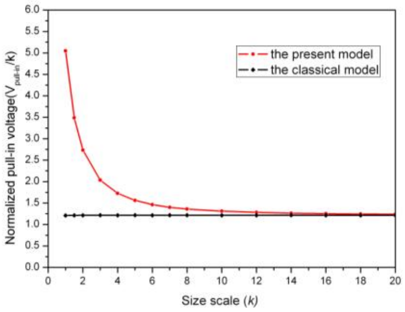

Figure 2 Size-dependent pull-in voltages

SIZE-DEPDENENT MICROPLATE MODEL

The governing equation of static deformation with electrostatically actuated microplate-based MEMS is also can be derived based on SGE theory and principle of minimum potential energy, which is expressed as

$$-p_1\nabla^6 w + p_2\nabla^4 w = \frac{\varepsilon V(t)^2}{2(d-w)^2} \tag{5}$$

in which

$$p_1 = I\mu(2l_0^2 + \frac{4}{5}l_1^2) \quad p_2 = \mu h(2l_0^2 + \frac{8}{15}l_1^2 + l_2^2) + (k + \frac{4}{3}\mu)I \tag{6}$$

$$\nabla^6 w = \frac{\partial^6 w}{\partial x^6} + 3\frac{\partial^6 w}{\partial x^4 \partial y^2} + 3\frac{\partial^6 w}{\partial x^2 \partial y^4} + \frac{\partial^6 w}{\partial y^6} \quad \nabla^4 w = \frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}$$

The boundary conditions for the size-dependent microplate are

$$w = 0; \quad \frac{\partial w}{\partial x} = 0; \quad \frac{\partial^4 w}{\partial x^4} = 0 \quad at\ x = 0, a \tag{7}$$

$$\alpha^2\frac{\partial^2 w}{\partial y^2} + \upsilon\frac{\partial^2 w}{\partial x^2} = 0; \ \alpha^2\frac{\partial^3 w}{\partial y^3} + (2-\upsilon)\frac{\partial^3 w}{\partial x^2 \partial y} = 0; \ \frac{\partial^4 w}{\partial y^4} = 0 \quad at\ y = 0, b$$

In which a and b are length and width of microplate, $\alpha = a/b$ the aspect ratio, υ the Poisson's ratio. When size effect is ignored, Eqs. (5) and (7) will reduces to ones of classical plate model.

The variation of normalized pull-in voltage ($V_{pull\text{-}in}/k$) with size scale (k) is presented in Fig. 2. The parameter k is defined as the ratio of plate thickness to MLSP: $k=h/Cl$, with h the plate thickness. And keeping the cross-sectional shape and aspect ratio of the plate to be the same, i.e. fixing a/h=200 and b/h=100. The gap is kept as g/h=1.Cl is taken to be $0.1\,\mu m$.Results predicted by the present model are compared with those predicted by the classical model in Fig. 2.

It is seen from Fig. 2 that the normalized pull-in voltage keeps a constant for the classical model, indicating no size effect. For the present size-dependent model, the pull-in voltage increases nonlinearly with the reduction of size scale. The two models show almost no difference for the normalized pull-in voltage if the size scale k is about larger than 15, indicating that the present model can be used in macro scale; while the present model shows strong size effect as the size scale further decreases, indicating that the classical model may be inadequate in micron scale or smaller. Fig. 2 again confirms that the present model may be regarded as an extension of the classical model, since the present model can be reduced to the classic one if the strain gradient tensors are ignored.

CONCLUSIONS

Size-dependent microbeam model and microplate model are developed for electrostatically actuated MEMS using strain gradient elasticity theory. Results show that the two new models can capture size effect of pull-in voltage and other properties. The new model may be robust for predicting the behavior of size-dependent pull-in instability for microbeam and microplate-based MEMS devices.

References

[1] Younis M.I., Abdel-Rahman E.M., Nayfeh A.: A reduced-order model for electrically actuated microbeam-based MEMS. *J. Microelectromech. Syst.*, **12**:672-680, 2003.

[2] Zhao X.P., Abdel-Rahman E.M., Nayfeh A.H.: A reduced-order model for electrically actuated microplates. *J. Micromech. Microeng.*, **14**:900-906, 2004.

[3] Lam D.C.C., Yang F., Chong A.C.M., Wang J., Tong P.: Experiments and theory in strain gradient elasticity. *J. Mech. Phys. Solids*, **51**:1477-1508, 2003.

[4] Tilmans H. A.C., Legtenberg R.:Electrostatically driven vacuum-encapsulated polysilicon resonators: 2. Theory and performance. *Sens. Actuators, A*: **45**: 67–84, 1994.

[5] Kuang J. H., Chen C. J.: Dynamic characteristics of shaped micro-actuators solved using the differential quadrature method. *J. Micromech. Microeng.* **14**: 647-655, 2004.