NONLOCAL VIBRATION OF NANOPLATE ON VISCOELASTIC FOUNDATION

S.A. Fazelzadeh, S. Pouresmaeeli
Department of Mechanical Engineering, Shiraz University, Shiraz 71348-51154, I.R. Iran

Summary Vibration of nanoplate resting on viscoelastic foundation is studied based on reformulating Kirchhoff's plate theory using the nonlocal elastic theory. Kelvin model is used for modelling the viscoelastic foundation. It is assume that ends of all sides are simply-supported. The complex frequency in closed-form expression is obtained in terms of the damping ratio. The influences of damping coefficient on the natural frequency of the nanoplate are also analyzed.

INTRODUCTION

Two-dimensional nanostructures like nanoplates reveal amazing mechanical and electrical properties [1]. Consequently nanoplates act very significant role in various nanotechnology industries like strain, mass and pressure sensors, energy storage, NEMS, solar cells, composite materials and etc. Unfortunately, due to the complexities in production of nanoplates inadequate number of researches have been reported on the nanoplates. For the sake of attaining applications of the nanoplates, investigation of these nanostructures has become an important issue. Therefore, the vibration characteristics of nanoplates have been studied by different researchers [2, 3]. In recent studies, nanoplates embedded in elastic foundation are studied. Then viscosity of foundation is not accounted. It is obvious that a more realistic model can be applied by insertion viscosity to the foundation model to consideration the effect of energy dissipation. In viscoelastic foundation, a portion of the energy of deformation is recoverable and the other portion of the energy is irrecoverable. Despite notable researches in the field of the vibrations of nanoplates, there has been no effort to tackle the problem expressed in the present paper. The subject of this paper is to study the vibration of single nanoplate resting on viscoelastic foundation using nonlocal elasticity theory. Viscoelastic foundation is modeled by Kelvin model. Kirchhoff's plate theory is reformulated using the nonlocal differential constitutive relations. Validation of the analytical model which is proposed here is done by comparing results for a nanoplate with those given in the literature. Finally, numerical results are presented to illustrate the effect of damping coefficient on the vibration of nanoplate.

FORMULATION

Nonlocal continuum theory [4] is expressed as

$$(1-\mu^2\nabla^2)\boldsymbol{\sigma}_{ij} = \mathbf{t}_{ij} \tag{1}$$

The terms $\boldsymbol{\sigma}_{ij}$ and $\mathbf{t}_{ij}$ are the nonlocal stress and classical stress, respectively. The Laplace operator in the Cartesian coordinate system is given by $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$. The parameter μ is the scale coefficient revealing the small-scale effect. Consider a nanoplate of length L in the x direction, width W in the y direction and thickness h. Because principal of virtual work is independent of constitutive relations, following governing equations can be used:

$$\frac{\partial^2 M_{xx}}{\partial x^2} + \frac{\partial^2 M_{yy}}{\partial y^2} + 2\frac{\partial^2 M_{xy}}{\partial x\partial y} + q = \rho h\frac{\partial^2 w}{\partial t^2} - \frac{\rho h^3}{12}\left(\frac{\partial^4 w}{\partial x^2\partial t^2} + \frac{\partial^4 w}{\partial y^2\partial t^2}\right) \tag{2}$$

where q is the distributed transverse load on the nonlocal plate and M_{xx}, M_{yy} and M_{xy} are stress couple resultants which are defined as

$$\{M_{xx}, M_{yy}, M_{xy}\} = \int_{\frac{-h}{2}}^{\frac{h}{2}} \{\sigma_{xx}z, \sigma_{yy}z, \sigma_{xy}z\}dz \tag{3}$$

Using Eq. (1) and Eq. (3), we can express stress resultants in terms of displacements as follows:

$$M_{xx} - \mu^2\nabla^2 M_{xx} = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right), \qquad M_{yy} - \mu^2\nabla^2 M_{yy} = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right), \qquad M_{xy} - \mu^2\nabla^2 M_{xy} = -D(1-\nu)\frac{\partial^2 w}{\partial x\partial y} \tag{4}$$

where D ($D = Eh^3/12(1-\nu^2)$) denotes the bending rigidity of the nanoplate. Moreover, E and ν denote Young's moduli and Poisson's ratio, respectively. A typical nanoplate resting on viscoelastic foundation is shown in Fig. 1. For mathematical modeling of the foundation, we use the Kelvin model [5]. The stiffness and damping of the springs are k and c, respectively.

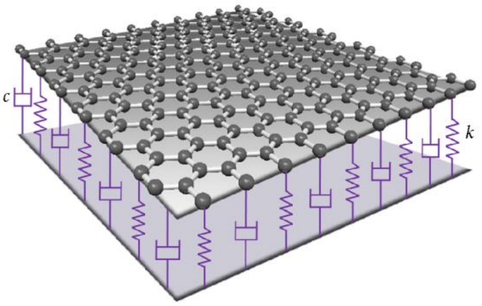

Fig 1. Schematic of nanoplate on viscoelastic foundation.

By using Eq. (2) and Eq. (4), the governing equation of motion in terms of the displacement is obtained as:

$$D\nabla^2(\nabla^2 w)+\rho h\frac{\partial^2 w}{\partial t^2}-\rho h\mu^2\nabla^2\frac{\partial^2 w}{\partial t^2}+kw-k\mu^2\nabla^2 w+c\frac{\partial w}{\partial t}-c\mu^2\nabla^2\frac{\partial w}{\partial t}=0 \tag{5}$$

It is assumed that all edges of the nanoplate are simply-supported. The boundary conditions are expressed as

$$w(0,y,t)=w(L,y,t)=w(x,0,t)=w(x,W,t)=M(0,y,t)=M(L,y,t)=M(x,0,t)=M(x,W,t)=0 \tag{6}$$

The governing equation can be solved by the Navier method assuming the solutions in the form

$$w=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{mn}\sin(\frac{m\pi}{L}x)\sin(\frac{n\pi}{W}y)e^{\lambda t} \tag{7}$$

where m and n are half wave numbers. Substituting Eq. (7) into Eq. (5), we obtain eigenfrequencies as follow.

$$\lambda_{1,2}=(\frac{-c}{2\rho h})\pm i\sqrt{(\frac{k}{\rho h}+D[(\frac{m\pi}{L})^2+(\frac{n\pi}{W})^2]^2\Big/\rho h[1+\mu^2((\frac{m\pi}{L})^2+(\frac{n\pi}{W})^2)])-(\frac{c}{2\rho h})^2} \tag{8}$$

NUMERICAL RESULTS

This model is justified by a good agreement between the results given by present model and available data in literature for the case of the square nanoplate (Fig. 2). In this case, we take E = 1.02 TPa, ν = 0.3, ρ = 2300 kg/m^3, h = 0.34 nm and μ = 1 nm [3]. Fig. 3 shows the variations in the frequencies (imaginary part) and decaying rates (real part) of nanoplate vibration, with damping coefficient for E = 1.02 TPa, ν = 0.3, ρ = 2300 kg/m^3, h = 0.34 nm, k = 10^{15} Pa/m and μ = 1 nm. Several interesting results can be seen from this figure. At c=0 the real parts of the eigenvalues are zero and the motion of the nanoplate is referred to as undamped vibration. With increasing c up to $c=2.05\times10^6$ Pa.s/m, the imaginary parts of the eigenvalues are diminished. It should also be noted that as c is further increased, the real parts of eigenvalue tend to infinity and zero.

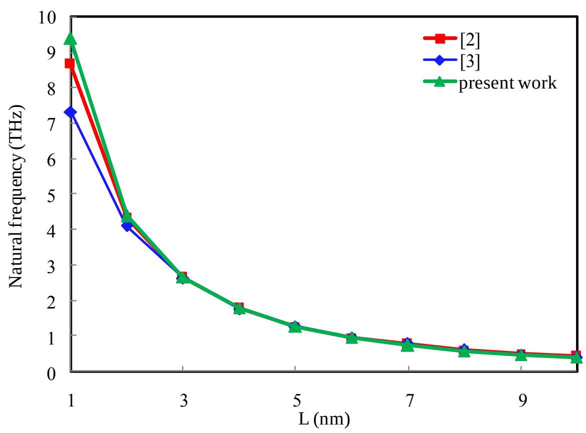

Fig. 2. Comparison between fundamental frequencies given by present model and available data in literature.

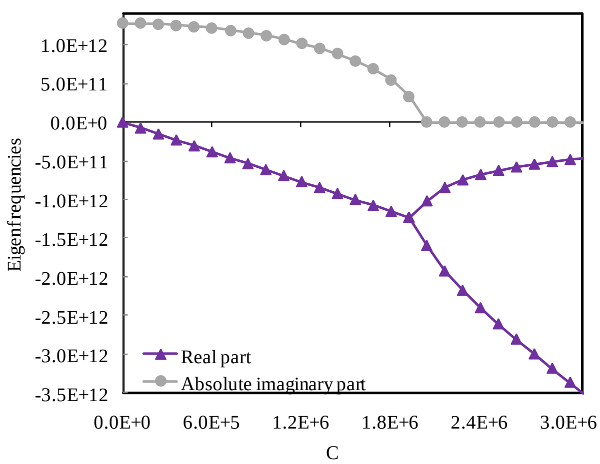

Fig. 3. Variations of the real and imaginary parts of eigenfrequencies with damping coefficient.

CONCLUSIONS

To illustrate the influence of damping coefficient on the vibration of nanoplate resting on viscoelastic foundation, analytical approach based on the Navier method has been developed. The validity of the suggested model was justified by comparing it with some available known data in literature. The numerical results showed that the dynamic behavior of the nanoplate resting on viscoelastic foundation is sensitive to the foundation damping. Therefore, this effect must be taken into account in design of nanocomposites.

References

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