A MODEL FOR THE ENERGY HARVESTING PERFORMANCE OF SHEAR MODE PIEZOELECTRIC CANTILEVER

Liang Zhou*, Jing Sun*, Xuejun Zheng**a), Jihe Zhao*, Shutao Peng* and Yong Zhang*
*Faculty of Materials, Optoelectronics and Physics, Xiangtan University, Xiangtan, Hunan 411105, P.R. China
**School of Materials Science and Engineering, University of Shanghai for Science & Technology, Shanghai 200093, P.R.China

Summary The electrical model with the piezoelectric constitutive equations of d_{15} mode and the single degree of freedom model are combined to describe the energy harvesting performance of shear mode piezoelectric cantilever, and the proposed model is used to simulate the frequency dependence of the output peak voltage and the output power. The analytical results are compared with the previous experimental results and the numerical solutions based on finite element method, which reveal acceptable agreement.

INTRODUCTION

Recently, piezoelectric energy harvesting has attracted considerable attention [1]. Different piezoelectric modes are adopted for piezoelectric energy harvesters. Using piezoelectric materials for energy harvesting, the shear type, d_{15} mode, might provide a larger output as compared to the d_{31} and d_{33} modes for certain loading cases [2]. B. Ren *et al.* fabricated the piezoelectric energy harvester based on d_{15} mode cantilever with $0.71Pb(Mg_{1/3}Nb_{2/3})O_3$-$0.29PbTiO_3$ (PMN-PT) single crystal, and it is of the largest power density among the three modes of piezoelectric energy harvesters [3]. D.A Wang et al. [4] introduced a shear mode piezoelectric energy harvester based on a pressurized water flow, and the numerical analysis is given to compare with the experiment. There are many analytical models on the energy harvesting piezoelectric cantilever with the d_{31} and d_{33} modes to investigate the energy harvesting performance [1,5], and the analytical model about d_{15} mode piezoelectric cantilever is required to investigate the energy harvesting performance.

SINGLE DEGREE OF FREEDOM MODELLING AND FEM ANALYSIS

A piezoelectric energy harvester based on shear mode cantilever with PMN-PT single crystal was proposed [3] as shown in Fig. 1. The harvester consists of a piezoelectric layer and a metal layer. The piezoelectric layer is poled along the length direction (x_3-axis). One end of the cantilever is fixed on the base to vibrate along the vertical direction harmonically with a magnitude Y_0 at a given angular frequency ω, and the other is connected with a proof mass M_{tip}. In this paper, combining the single degree of freedom (SDOF) model, an electrical model with the piezoelectric constitutive equations of d_{15} mode is proposed to describe the energy harvesting performance of shear mode piezoelectric cantilever. The equivalent SDOF system is described as [6]

$$\ddot{z} + 2\zeta\omega_n \dot{z} + \omega_n^2 z = -\ddot{y}, \quad (1)$$

where z is the relative motion between the seismic mass and the vibrating base, ζ the damping ratio and ω_n the resonance angular frequency. For the electrical model, the piezoelectric constitutive equations of d_{15} mode can be expressed as [3]

$$E_1 = -h_{15}S_5 + \beta_{11}^S D_1, \quad (2)$$

where E_1 and D_1 are the electric field and electric displacement along the thickness direction, h_{15} the shear mode piezoelectric constant, and β_{11}^S the dielectric impermittivity. E_1 and D_1 can be expressed as $E_1 = U/t_P$ and $D_1 = Q/A$, where A, U and Q are the electrode area, the voltage and charge. The voltage can be obtained by substituting the expressions of E_1, D_1 and S_5 into Eq. (2)

$$U = R_L\dot{Q} = R_L C_P \dot{U} + \frac{R_L A h_{15} K L_P (3L - L_P)}{12\beta_{11}^S EI}\dot{z}, \quad (3)$$

where C_P is the capacitance of piezoelectric layer, and R_L is the external resistance. Combining Eq. (1) and Eq. (3), the amplitude of the voltage induced by the mechanical vibration can be obtained as follows

$$U_P = \frac{\omega A e_{15} K L_P (3L - L_P)}{12EI} \frac{\ddot{Y}_0/\omega_n^2}{\sqrt{(\omega/\omega_n)^4 + 2(2\zeta^2 - 1)(\omega/\omega_n)^2 + 1}} \Big/ \left[(1/R_L)^2 + (\omega C_P)^2\right]^{1/2} \quad (4)$$

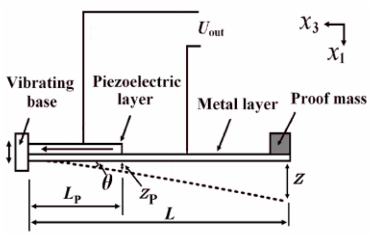

Fig. 1 The shear mode piezoelectric cantilever

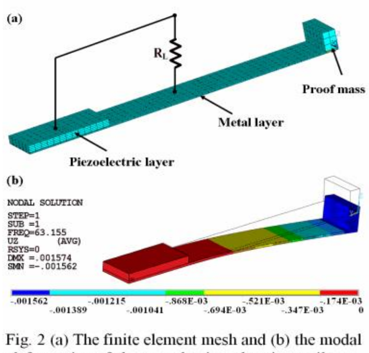

Fig. 2 (a) The finite element mesh and (b) the modal deformation of shear mode piezoelectric cantilever.

The output peak voltages of the shear mode energy harvester are also simulated by numerical method using ANSYS, and Fig. 2 shows the finite element mesh and modal deformation. The coupled-field element SOLID5 is used for the piezoelectric material, the structural element SOLID45 is used for the nonpiezoelectric material

a) Corresponding author. E-mail: zhengxuejun@usst.edu.cn; zhengxuejun@xtu.edu.cn

and the piezoelectric circuit element CIRCU94 is used for the load resistor [7]. The output peak voltages versus excitation frequencies can be obtained through harmonic analysis. The numerical results are compared with the analytical results to verify the proposed model.

RESULTS AND DISCUSSION

The frequency dependence of output peak voltage for the experimental load resistances R_L=1000, 667, 444 and 231 KΩ is simulated to compare with that of the shear mode piezoelectric cantilever with PMN-PT single crystal and brass shim [3], and the model predictions and experimental measurements are given in Fig. 3. The ANSYS numerical solutions are also given to be compared. The resonance frequency of 63.8 Hz is obtained by analytical simulation, which is close to that of 60 Hz in experiment and that of 63.2 Hz obtained by numerical modal analysis as shown in Fig. 3. The output peak voltage increases at the range of 0-63.8 Hz and decreases at 63.8-110 Hz dramatically, while it is stable at 150-250 Hz. From Figs. 3(a)-(d), the output peak voltage increases with the load resistance, because the load resistance is parallel to the piezoelectric element regarded as a current source according to Eq. (4). The output peak voltage versus frequency curves simulated by proposed model are very close to the experimental and numerical results, indicating that the proposed model is effective to predict the frequency dependence of output peak voltage for the shear mode piezoelectric cantilever.

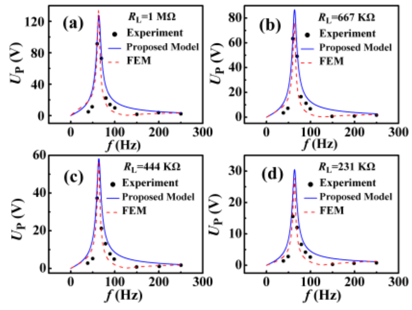

Fig.3 The model predictions, experimental measurements [3] and numerical results about peak voltage versus frequency

The frequency dependence of output power at the load resistances 1000, 667, 444 and 231 KΩ is respectively plotted in Figs. 4(a)-(d) for the shear mode piezoelectric cantilever. The output power can be calculated by $P = \frac{U_P^2}{2R_L}$. As the frequency increases, the variation trends of the output power with frequency are similar to those of output peak voltage in Fig. 4. The higher output power is obtained with larger load resistance, indicating R_L=1 MΩ is close to the matching load resistance of piezoelectric layer. The output power of 8.14 mW with R_L=1 MΩ at resonance frequency can be applied to low-power portable electronics and wireless sensors. The output power versus frequency curves simulated by proposed model are close to the experimental and numerical *P*-*f* curves, indicating that the proposed model is effective for simulating the output power of the shear mode piezoelectric cantilever.

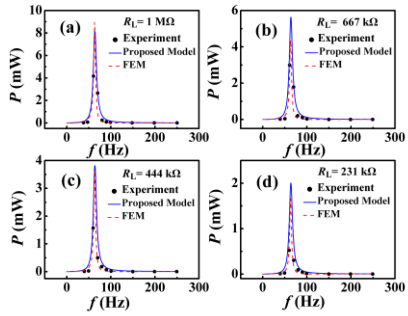

Fig.4 The model predictions, experimental measurements [3] and numerical results about the power versus frequency

CONCLUSIONS

In summary, combining the SDOF model with the piezoelectric constitutive equations of d_{15} mode, we proposed the analytical model to analyze the energy harvesting performance of shear mode piezoelectric cantilever. The validity of the proposed model is verified, because the simulated curves are very close to the experimental and numerical results of output peak voltage and output power at the different frequencies. The model can successfully predict the coupled electrical and mechanical responses of the piezoelectric cantilever. According to the proposed analytical model we can choose the optimal piezoelectric material and geometrical dimensions to achieve not only the maximum electrical output responses but also the ideal resonance frequency in the application of the piezoelectric energy harvester.

References

[1] Erturk A., Inman D.J.: An experimentally validated bimorph cantilever model for piezoelectric energy harvesting from base excitations, *Smart Mater. Struct.* **18**:025009, 2009.

[2] Edery-Azulay L., Abramovich H.: Piezoelectric actuation and sensing mechanisms —closed form solutions, *Composite Structures* **64**: 443-453, 2004.

[3] Ren B., Or S.W., Zhang Y.Y., Zhang Q.H., Li X.B., Jiao J., Wang W., Liu D.A., Zhao X.Y., Luo H.S.: Piezoelectric energy harvesting using shear mode $0.71Pb(Mg_{1/3}Nb_{2/3})O_3$–$0.29PbTiO_3$ single crystal cantilever, *Appl. Phys. Lett.* **96**: 083502, 2010.

[4] Wang D.A., Liu N.Z., A Shear Mode Piezoelectric Energy Harvester Based on a Pressurized Water Flow, *Sensors Actuators* A. **167**:449-458, 2011.

[5] Mo C., Kim S., Clark W.W.: Theoretical analysis of energy harvesting performance for unimorph piezoelectric benders with interdigitated electrodes, *Smart Mater. Struct.* **18**:055017, 2009.

[6] Ng T.H., Liao W.H.: Sensitivity analysis and energy harvesting for a self-powered piezoelectric sensor, *Journal of Intelligent Material Systems and Structures* **16**:785-797, 2005.

[7] Zhu M., Worthington E., Njuguna J.: Analyses of power output of piezoelectric energy-harvesting devices directly connected to a load resistor using a coupled piezoelectric-circuit finite element method, *IEEE T Ultrason Ferr.* **56**:1309-1318, 2009.