Experimental and Numerical Study on Load Sharing Behavior of Gear System Actuated by Multiple Pinions Simultaneously

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Summary The dynamic model and the state-space equations of the gear system actuated by multiple pinions are established. A new load sharing index is defined based on ratio of the minimum and maximum of Floquet multipliers. The numerical simulation and experiment has been investigated for an example gear system driven by two pinions. The results show that this load sharing index can evaluate the load transmission behavior of multiple mesh gear system effectively.

DYNAMIC MODEL OF GEAR SYSTEM

A parallel arrangement of multiple pinions is widely utilized in various heavy-duty industrial applications to generate large torques. The load sharing performance, i.e. every mesh gear pair in the system has the identical load, is very important for the dynamic behavior and fatigue life of gear system because the synchronous control system is usually used for the drives of multiple pinions as well as the pinions have the same velocity. Sometimes, the coupling effect between the non-uniformed load transmission of every mesh gear pair and the synchronous control of multiple pinions may lead to the wear and failure of gear system. As a consequence, it is necessary to investigate the load sharing performance of multiple mesh gear system in terms of its intrinsic characteristics.

A lumped parameter model of multiple gear pair system in which the central gear is actuated by many surrounding pinions simultaneously is shown in Fig.1. The dynamic equations of motion of the multiple gear system can be written as

$$\begin{gathered} I_{pi}\ddot{\theta}_{pi} + c_i R_{pi}\left(R_{pi}\dot{\theta}_{pi} - R_g\dot{\theta}_g\right) + k_i(t)R_{pi}\left(R_{pi}\theta_{pi} - R_g\theta_g\right) = T_{pi} \\ I_g\ddot{\theta}_g - R_g\sum c_i\left(R_{pi}\dot{\theta}_{pi} - R_g\dot{\theta}_g\right) - R_g\sum k_i(t)\left(R_{pi}\theta_{pi} - R_g\theta_g\right) = -T_g \end{gathered} \tag{1}$$

where the i stands for the i th pinion and $i = 1,\cdots,n$. The moments of inertias of the i th pinion and the gear are I_{pi} and I_g. The radii of base circles of the i th pinion and the gear are R_{pi} and R_g, $\ddot{\theta}_{pi}$, $\dot{\theta}_{pi}$ and θ_{pi} are the angular acceleration, the angular velocity and the rotating angle of the i th pinion, $\ddot{\theta}_g$, $\dot{\theta}_g$ and θ_g are the angular acceleration, the angular velocity and the rotating angle of the gear, T_{pi} and T_g are the torque moments acted on the i th pinion and the gear, and c_i and $k_i(t)$ are the damping coefficient and meshing stiffness of the i th gear pair. If the dynamic transmission error of the i th pinion and the gear is assumed to be $q_i = R_{pi}\theta_{pi} - R_g\theta_g$, the number of the equations in Eq. 1 can be reduced from $n+1$ to n. The stability of system is determined by the continuous time-variant state-space equation

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) \tag{2}$$

where $\mathbf{x} = [q_1, q_2, \cdots, q_n, \dot{q}_1, \dot{q}_2, \cdots, \dot{q}_n]^T$ is the state vector and $\mathbf{A}$ is the state matrix. In terms of the Floquet theory, the fundamental matrix $\boldsymbol{\varphi}(t)$ is comprised of a complete set of fundamental solutions and $\dot{\boldsymbol{\varphi}}(t+T) = \mathbf{A}(t)\boldsymbol{\varphi}(t+T)$. Here, t and T are the mesh time and the mesh period, respectively. Since the solution matrix is obtained in terms of the fundamental matrix, there exists a matrix $\mathbf{C}$ which satisfies $\boldsymbol{\varphi}(t+T) = \boldsymbol{\varphi}(t)\mathbf{C}$. If $\mathbf{B} = \ln\mathbf{C}/T$, we can obtain $\boldsymbol{\varphi}(t+T) = \boldsymbol{\varphi}(t)e^{T\mathbf{B}}$. Then, $e^{T\mathbf{B}} = \boldsymbol{\varphi}^{-1}(t_0)\boldsymbol{\varphi}(t_0+T)$ is called the monodromy matrix. The eigenvalues of the monodromy matrix, λ_i, are called the Floquet multipliers of the system. The ratio of state vector (dynamic transmission error) at some time t may be obtained as

$$\left|x_i(t)/x_j(t)\right| = \left|\lambda_i/\lambda_j\right| \tag{3}$$

The load sharing index (LSI) can be defined as the ratio of the minimum and maximum of the Floquet multipliers. The range of LSI is from 0 to 1. The load sharing behavior becomes worse when LSI approaches to 0. In this case, the mesh force of some gear pair is bigger than that of others, which is dangerous for the damage and failure of structures. On the other hand, when LSI approaches to 1, it means that the load sharing behavior becomes better. The load on every mesh gear pair is almost identical.

$$\mathbf{LSI} = \min\left|\lambda_i\right| / \max\left|\lambda_i\right| \qquad (i = 1,\cdots,n) \tag{4}$$

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RESULTS AND DISCUSSIONS

A three degree of freedom gear pair system can be obtained if two pinions are used to actuate a gear shown in Fig.1b. The differential equation of motion is deduced in terms of the Newton-Euler principles. In the testing apparatus, the pinion is driven by electric motor. The mounted locations of motors in the testing apparatus are shown in Fig.1a. The external torque is applied on the gear. The second motor is located at 270° relative to horizontal axis. There are two mounted locations for the first motor, 180° and 60° shown in Fig. 1a. The mechanical parameters of numerical model are the same as that of apparatus. The simulation was done in Matlab2009.

The correlations of torque on the motor shafts and time are illustrated in Fig. 2. The numerical results are in agreement with the experimental data. In the first case (Fig.2a), the LSI in terms of Floqeut multipliers is 0.8088. The ratio of maximal torques of the two pinions is about 0.801. It means that the load sharing behavior is better when the two pinions are mounted in 270° and 180°. In the second case (Fig. 2b), the load sharing index is 0.526. The ratio of maximal torques of the two pinions is about 0.4. The vibration of torque on motor shaft becomes dramatic. The difference of torque on the two motor shafts becomes larger. It is hardly for the second motor to provide enough torque for the entire gear system. The load sharing performance is obviously worse than that in the first case. The evaluation of LSI is in consistent with the results from the experiment.

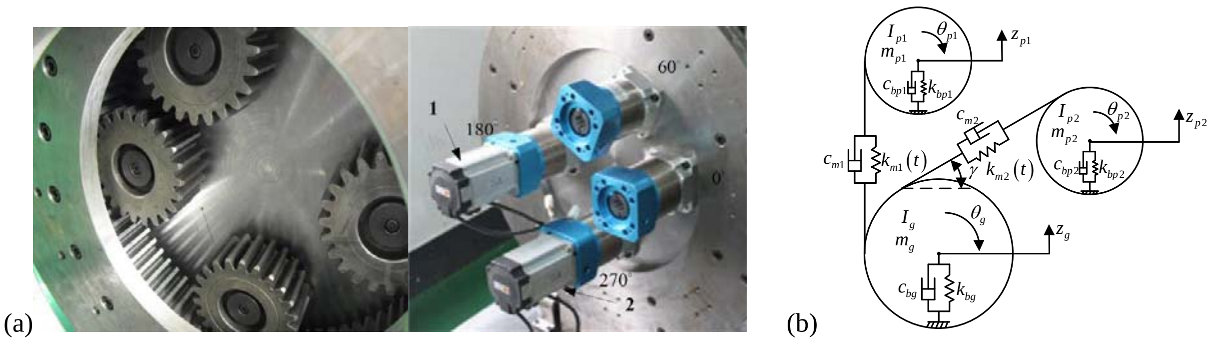

Fig.1. A lumped parameter model of multiple pinion drives

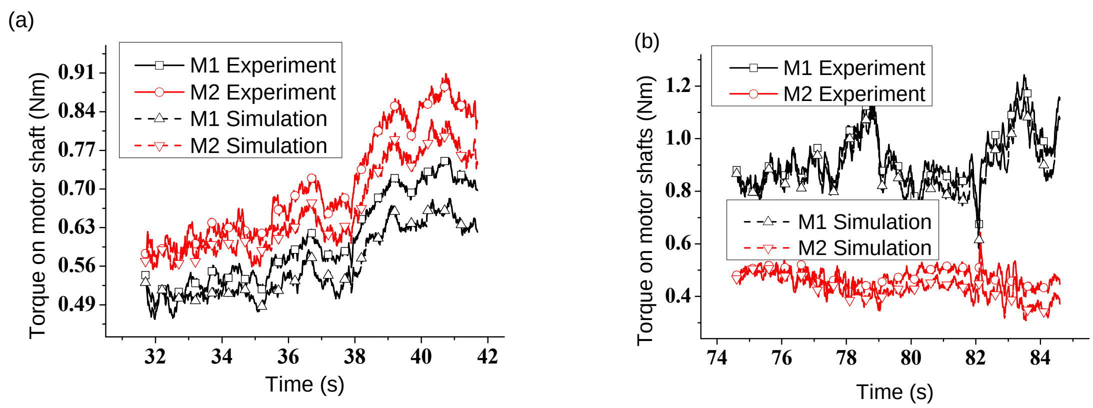

Fig.2. Correlation of torque on motor shafts and time. M1 stands for motor1, M2 stands for motor2, (a) (180°, 270°), (b) (60°, 270°)

CONCLUSIONS

The dynamic model and state space equations of gear system actuated by multiple pinions simultaneously are established. The new load sharing index is proposed. The mounted locations of pinions are dependent upon the load transmission behavior in the multiple mesh gear system. The new load sharing index can evaluate the load transmission behavior of various mesh gear pairs according to the comparison of numerical results and experimental data. It is useful in the structure design of multiple mesh gear pair system.

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