ROBUST DESIGN OF AN ALL-WHEEL STEERING CAR

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Summary The aim of the paper is to improve drivability of passenger cars by adding a proper rear steering strategy. This strategy is optimized for uncertain car and tire parameters where the objectives concerning lateral acceleration and yaw velocity are formulated in the frequency domain. Due to the uncertainty the objectives are random variables. Therefore, mean values are minimized to obtain best mean behavior for all driving conditions, and variances are minimized to achieve best robustness against change of car parameters. Comparison with a reference car shows improved results for both criterion values and probability distributions.

VEHICLE MODEL WITH REAR STEERING STRATEGY AND UNCERTAINTIES

The vehicle is represented by a single track model based on [1], see Fig. 1. The tire-side forces F_f and F_r are modeled as PT_1 behavior [2], and the rear wheel steering angle δ_r is modeled as PDT_1 behavior

$$T_r\,\dot{\delta}_r + \delta_r = \frac{i_r}{i_f}\,\delta_H + \frac{i_{vr}T_r}{i_f}\,\dot{\delta}_H \tag{1}$$

where the front wheel steering angle δ_f follows the hand wheel steering angle δ_H according to $\delta_f = \delta_H/i_f$ with steering ratio i_f. According to [3] the equations of motion for the vehicle, tire and control can be summarized in the state equations

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(v,\boldsymbol{p}_v)\,\boldsymbol{x} + \boldsymbol{b}(\boldsymbol{p}_v)\,\dot{\delta}_H, \quad \boldsymbol{x} = \begin{bmatrix}\beta & \dot{\psi} & F_f & F_r & \delta_H & \delta_r\end{bmatrix}^T, \tag{2}$$

which depend on drive speed v, speed dependent control parameters and adjustable vehicle parameters summarized in

$$\boldsymbol{p}_v = \begin{bmatrix} i_f(v), & i_r(v), & i_{vr}(v), & T_r(v), & l, & S_f, & S_r, & c_f, & c_r, & m, & l_f, & J_z\end{bmatrix}^T. \tag{3}$$

In order to reduce computational effort of solving differential equation (2), it is transformed into the frequency domain. With $s = i\omega$ we get the transfer functions

$$\boldsymbol{H}(\omega; v, \boldsymbol{p}_v) = \begin{bmatrix} H_\beta & H_{\dot{\psi}} & H_{F_f} & H_{F_r} & 1 & H_{\delta_r}\end{bmatrix}^T := \boldsymbol{X}/\Delta_H = \left(i\omega\boldsymbol{E} - \boldsymbol{A}(v,\boldsymbol{p}_v)\right)^{-1}\boldsymbol{b}(\boldsymbol{p}_v)i\omega \tag{4}$$

relating the *LAPLACE*-transformed state $\boldsymbol{X}(s) = \mathcal{L}\{\boldsymbol{x}(t)\}$ to steering wheel angle $\Delta_H(s) = \mathcal{L}\{\delta_H(t)\}$. For assessing drive quality we additionally need the lateral acceleration which results from Fig. 1 as $\ddot{y} = v^2/\rho = v^2/[v/(\dot{\beta}+\dot{\psi})] = v(\dot{\beta}+\dot{\psi})$. *LAPLACE* transformation yields the transfer function $H_{\ddot{y}} = v(i\omega H_\beta + H_{\dot{\psi}})$.

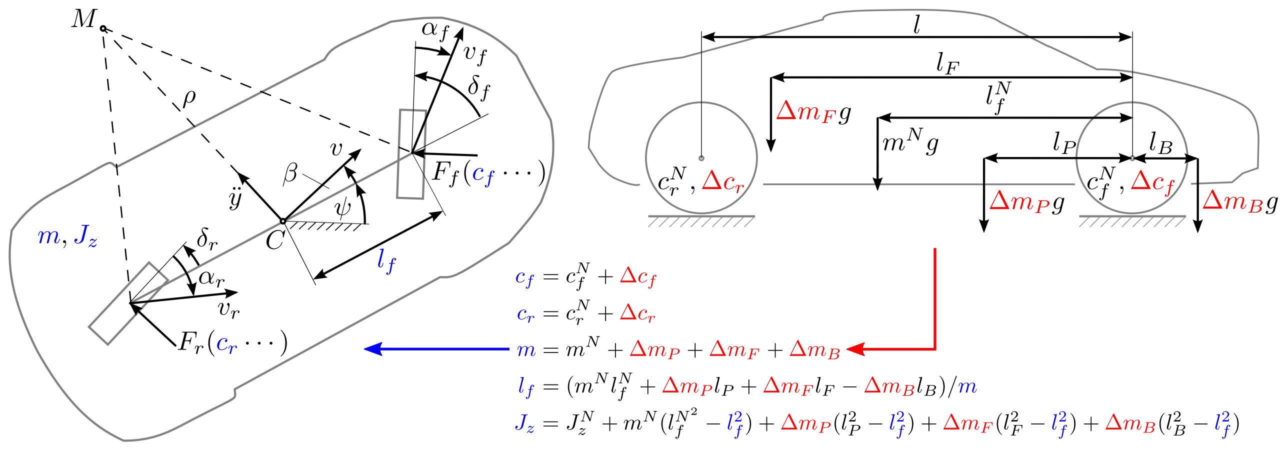

Figure 1. Single track model with uncertainties

Optimal car design requires a proper choice of values for the parameters and functions (3). In [3] the speed dependent functions are parameterized by Bézier splines and numerical optimization is performed resulting in optimal values for the design variables like vehicle's mass geometry m, J_z, l_f or tire cornering stiffnesses c_f, c_r. However, in reality the latter will be uncertain due to fabrication tolerances, and the mass geometry is uncertain due to unknown number of passengers, baggages, fuel level, etc., Fig. 1. Thus, for robust design the certain parameters like the steering parameters of (1) and nominal parameters for mass geometry and tire cornering stiffnesses may be summarized in a design vector $\boldsymbol{p}$, and the uncertainties (red colored) in Fig. 1 are summarized in $\boldsymbol{u}$, where additional masses are considered as mass points at prescribed locations. The actual car behavior will then be determined by the actual values of the car parameters $\boldsymbol{p}_v = \boldsymbol{p}_v(\boldsymbol{p}, \boldsymbol{u})$.

FORMULATION OF THE ROBUST DESIGN PROBLEM

The major design objectives concerning drive quality and drive agility are the amplitudes of frequency responses

$$a_j(\omega; v, \boldsymbol{p}_v) := |H_j(\omega; v, \boldsymbol{p}_v)| \qquad \text{where} \qquad j \in \{\dot{\psi}, \ddot{y}\}\,, \tag{5}$$

(green lines in Fig. 2a). More precisely, the deviation from a desired frequency behavior (black dashed lines in Fig. 2a) is averaged over an interesting frequency interval and velocity range, and normalized with respect to a given nominal reference car (black solid lines in Fig. 2a), resulting in two objectives $f_{\ddot{y}}(\boldsymbol{p}_v)$ and $f_{\dot{\psi}}(\boldsymbol{p}_v)$ to be minimized. Due to the uncertain parameters $\boldsymbol{p}_v(\boldsymbol{p}, \boldsymbol{u})$ these objectives are random variables as well. Therefore, mean values $\bar{f}_{\ddot{y}}$, $\bar{f}_{\dot{\psi}}$ are determined to assess the mean drive quality, and variances $\mathrm{v}_{\ddot{y}}$, $\mathrm{v}_{\dot{\psi}}$ assess the robustness of a given design. This results in the multi-objective optimization problem

$$\min_{\boldsymbol{p}} \boldsymbol{f}(\boldsymbol{p}) \;\; \text{where } \boldsymbol{f} = \left[\bar{f}_{\ddot{y}}, \quad \bar{f}_{\dot{\psi}}, \quad \mathrm{v}_{\ddot{y}}, \quad \mathrm{v}_{\dot{\psi}}\right]^T. \tag{6}$$

Additionally the amplitude frequency responses of nominal designs without uncertainties should not exceed the maxima of the nominal reference car (red lines in Fig. 2a) by more than a user-defined relative tolerance $\varepsilon \ll 1$, respectively. These inequality constraints are added to the objectives (6) by a proper penalty strategy similar to [3] yielding penalized objectives

$$\boldsymbol{f}^\sigma = \left[\bar{f}^\sigma_{\ddot{y}}, \quad \bar{f}^\sigma_{\dot{\psi}}, \quad \mathrm{v}^\sigma_{\ddot{y}}, \quad \mathrm{v}^\sigma_{\dot{\psi}}\right]^T. \tag{7}$$

OPTIMIZATION AND DISCUSSION OF RESULTS

The objectives (7) are minimized simultaneously with the multi-objective genetic algorithm NSGA-II. In order to save computational time, the mean values and variances are not computed directly, but the objectives (5) are approximated by response surfaces, where Kriging models based on 30 support points are used. The statistics is then performed on these analytical surrogate models with 10^5 sample points chosen by optimal Latin hypercube sampling for the uncertainties $\boldsymbol{u}$. After the optimization process the non-dominated solutions are re-evaluated at 200 sampling points with the original single track model, checking differences in the solutions, and thus validating the response surface method. Fig. 2b shows all non-dominated solutions in the criterion space supplemented by the reference design R and deterministically optimized design D. Designs with best mean performance are located closest to the lower-left corner, whereas most robust designs are colored dark. Fig. 2c shows statistics of the non-dominated design P compared to deterministic optimum D and reference car R. Obviously the deterministic optimum (red) has a better mean performance than the reference car (black) with respect to both objectives. However, especially for the second criterion the histogram is much broader reflecting less robustness. Both, mean performance and robustness can be improved a little more by using the robust design concept proposed resulting e.g. in the shown design P (blue), however, it should be noted that due to time restrictions the optimization has not fully converged.

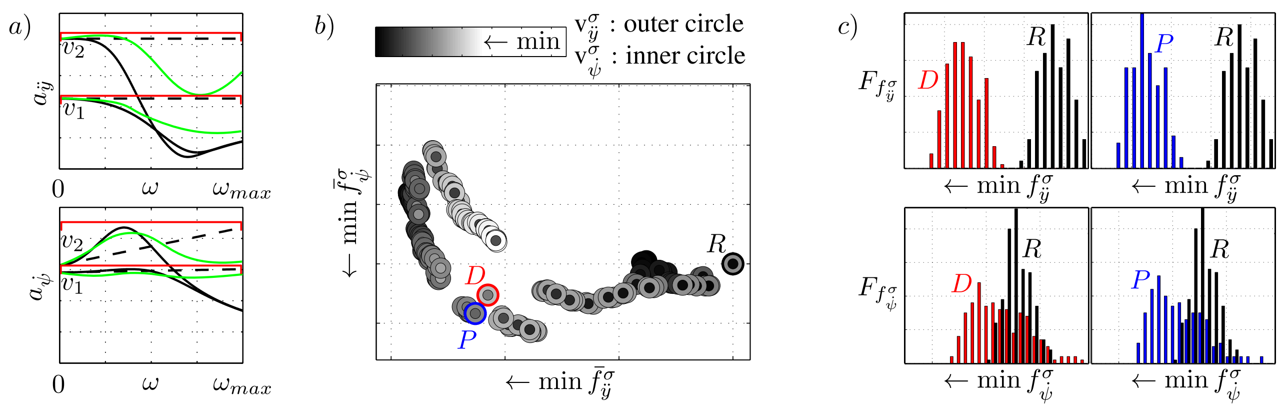

Figure 2. Robust optimization task and results: (a) optimization task in frequency domain (b) non dominated results in criterion space (c) exclusive frequency distributions

References

[1] M. Canale and L. Fagiano (2009) Comparing rear wheel steering and rear active differential approaches to vehicle yaw control, Vehicle System Dynamics, 48, pp. 529-546.

[2] M. Mitschke (2003) Dynamik der Kraftfahrzeuge, Springer, Berlin.

[3] J. Busch and D. Bestle (2011) Optimization of a new steering strategy to improve the driving behavior of cars, ENOC 2011, Rome, Italy.

[4] P. Flassig (2011) Unterstützende Optimierungsstrategien zur robusten aerodynamischen Verdichterschaufelauslegung, Shaker, Aachen.