ABOUT CONTROL POSSIBILITY OF A SATELLITE ATTITUDE

Zaure Rakisheva[*]
[*]*Department on Mechanics, Faculty of Mechanics and Mathematics, al-Farabi Kazak National University, Almaty, 050040, Kazakhstan*

Summary The problem of a satellite attitude control is considered. Satellite is considered as a solid body in the Newton field of forces. Motion is described by the system of Euler-Poisson equations. By original change of variables this system is reduced to the normal form with explicitly expressed linear part. The system solution is accepted as the non-perturbed motion and it is investigated on stability on first approximation. Control moments are found for obtaining of asymptotically steady motion in general case.

ANALYSIS OF CONTROL POSSIBILITY

One of the widespread mathematical models for describing of a satellite motion is a solid body with one fixed point, placed in the field of some forces (gravity, magnetic, Sun and Moon gravity, solar radiation pressure and so on). We will consider the motion in the Newton field of forces. Let *R* is the distance between the body's fixed point *O* and the centre of gravity O^*. We combine with the fixed point *O* the origins of the immovable coordinate system *OXYZ* with the axis *OZ*, directed along the vector $\overline{R} = \overline{OO^*}$, and of the mobile coordinate system $Oxyz$ with the axes, directed along the body's main axes of inertia. The solid body motion around the fixed point *O* is described by the system of the Euler-Poisson equations [1]. If distance *R* is great in comparison with the body sizes, the force function *U* may be presented in infinite series. As first two terms of series have the significant influence on motion, we take:

$$U(\alpha,\beta,\gamma) = -\frac{\lambda M}{R^2}(a\alpha + b\beta + c\gamma) - \frac{3}{2}\frac{\lambda}{R^3}(A\alpha^2 + B\beta^2 + C\gamma^2) + \ldots, \tag{1}$$

where α,β,γ are the direct cosines of the mobile axes with axis *OZ*; *M* is the satellite's mass; *a*, *b*, *c* are the coordinates of the satellite centre of mass concerning the mobile coordinate system. In this case the satellite motion is described by the following system of non-linear equations:

$$\begin{aligned} A\frac{dp}{dt} + (C-B)\,qr &= Mg(c\beta - b\gamma) + 3\frac{g}{R}(C-B)\beta\gamma, \\ B\frac{dq}{dt} + (A-C)\,rp &= Mg(a\gamma - c\alpha) + 3\frac{g}{R}(A-C)\gamma\alpha, \\ C\frac{dr}{dt} + (B-A)\,pq &= Mg(b\alpha - a\beta) + 3\frac{g}{R}(B-A)\alpha\beta. \\ \dot{\alpha} = r\beta - q\gamma;\ \ \dot{\beta} &= p\gamma - r\alpha;\ \ \dot{\gamma} = q\alpha - p\beta, \end{aligned} \tag{2}$$

where p,q,r are the projections of an angular speed of the body rotation on the mobile axes; *A*, *B*, *C* are the body's main moments of inertia; $g = \frac{\lambda}{R^2}$ is an acceleration of the gravitation force on distance *R*.

Applying the linear change of variables ($p,q,r,\alpha,\beta,\gamma$) $\to$ (u_1,u_2,u_3,u_4,u_5,u_6), suggested in [2], we may reduce the equations (2) to the system of the non-dimensional equations with explicitly expressed linear part:

$$\dot{u} = \widetilde{A}u + F(u), \tag{3}$$

where $\overline{u} = \{u_1,\ldots,u_6\}$ is the column vector; $\widetilde{A}$ is the constant square matrix of the 6th order; $\overline{F}(u)$ is the 6th order vector function, consisting of non-linear members of equations.

Studying this system on stability on the first approximation, we will obtain two zero characteristic roots and the biquadrate equation:

$$\lambda_1 = \lambda_2 = 0, \qquad \lambda^4 + N\lambda^2 + P^2 = 0.$$

Here *N* and P^2 are some constants, defined in [2]. The roots of the biquadrate equation depend on sign of its discriminant. If it is nonnegative, all the characteristic roots have zero real parts, and we cannot make conclusion about stability of the unperturbed motion. If it is negative, the biquadrate equation has two roots with positive real parts and according to the Lyapunov theorem [3] the unperturbed motion is unstable in this case.

For stabilization of unstable motion in the last case let us add to the right part of the equations of the system (4) the terms of the type $\mu K u_i$, $i = 1,\ldots,6$, where $\mu < 1$, $K = -\sqrt{2P - N}$. We will obtain the following system of the differential equations:

$$\dot{u} = \widetilde{B}u + F(u), \tag{4}$$

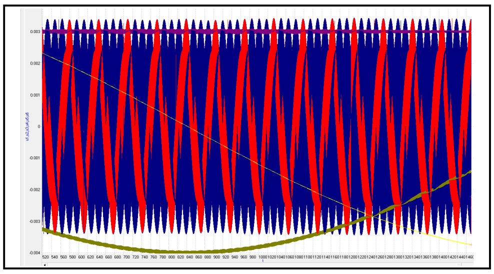
where $\tilde{B} = \tilde{A} + \mu K E$ is the constant square matrix and E is the unit matrix.
The system of first approximation gives the following characteristic roots:

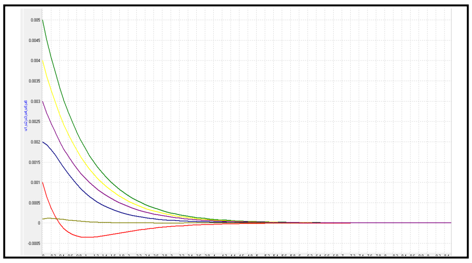
$$\lambda_1 = \lambda_2 = \mu K, \qquad \lambda_{3,4} = \frac{-\sqrt{2P-N}\,(2\mu - 1) \pm \sqrt{2P+N}\,i}{2}, \qquad \lambda_{5,6} = \frac{-\sqrt{2P-N}\,(2\mu + 1) \pm \sqrt{2P+N}\,i}{2}$$

Constraining μ within the limits $0{,}5 < \mu < 1$, we will obtain the negative real parts for all the roots of the characteristic equation. Hence, the Lyapunov's theorem about asymptotical stability of the unperturbed motion of the system (4) is true [3]. The additive control moments are created by the dissipative forces.
In case when the discriminant is nonnegative, the proposed method can be applied too. But there we have to constrain μ within the limits $-1 < \mu < 0$ and we will obtain the negative real parts for all the roots of the characteristic equations. Hence, the Lyapunov's theorem about asymptotical stability of the unperturbed motion of the system (4) by the above-stated restrictions is true.

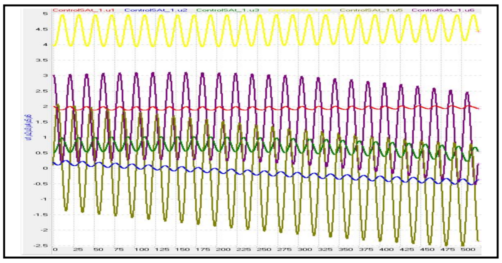
NUMERICAL CALCULATIONS

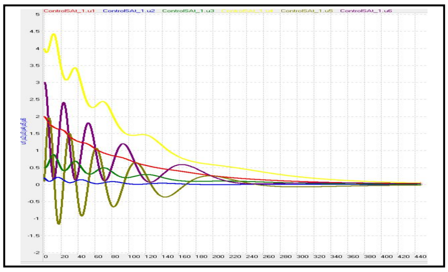
The results of theory were checked by numerical calculations. The geostationary satellite with mass 2,5 ton and moments of inertia $A = B = 1{,}2 \cdot 10^4, C = 5 \cdot 10^3\,(\text{kg}\times\text{m}^2)$ was considered. For this satellite the transformed system of equations was solved numerically without and with controlling moments.





As the second example the small spacecraft was considered with mass 250 kg, moments of inertia $A = 65, B = 90, C = 75\,(\text{kg}\times\text{m}^2)$ and altitude of 750 km. Figures show the calculated functions $u_1,\ldots u_6$ depending on time without and with controlling moments.





As both examples belong to the case with the positive discriminant, μ is constrained within the limits $-1 < \mu < 0$. Figures show that all variables reach steady state very quickly depending on the magnitude of parameter μ .

CONCLUSIONS

One way of the satellite attitude control was considered. With a help of linear change of the variables we reduced the initial system to the system of the non-dimensional equations with explicitly expressed linear part. The transformed system of the equations was investigated on stability on the first approximation. It was shown that it is enough to add small dissipative forces, which create moments, stabilizing the body's attitude. The results were corroborated by numerical calculations.

References

[1] Arkhangelsky Ju.A.: Analytical dynamics of the solid body. Nauka, Moscow 1977.

[2] Rakisheva Zaure: Motion stabilization of a solid body with fixed point. Progress in analysis and its applications. Proceedings of the 7th International ISAAC Congress: 636--642, World Sci. Publ., Hackensack, NJ, 2010.

[3] Malkin I.G.: Theory of the motion stability. Nauka, Moscow 1966.