IMPULSIVE SYNCHRONIZATION MOTION IN NETWORKED MULTIBODY SYSTEMS

Xingjie Wu*, **, Jin Zhou*[a)]

*Shanghai Institute of Applied Mathematics and Mechanics, Shanghai University, Shanghai 200072, China

**Department of Mathematics, Hefei Normal University, Hefei Anhui 230061, China

Summary This present paper investigates impulsive synchronization motion of networked multibody systems. Impulsive motion occurs when the networked systems are physically subject to either direct or indirect impulsive effects, or when subjected to both simultaneously. The governing equations of networked multibody systems are developed from Lagrange formulation. The investigation automatically incorporates a preliminary feedback control and the effects of impulsive constraints through its analysis. Some generic criteria on exponential synchronization of the system output with respect to generalized coordinates and its velocities over, respectively, undirected fixed is derived analytically. An example and simulations are used to demonstrate and validate the analysis procedure.

INTRODUCTION

Inspired by recent research advances in coordinated behavior and cooperative control of multi-agent systems, as well as its broad applications, synchronization motions of networked mechanical systems modeled by Lagrange dynamics have recently attracted a great deal of attention [1-3]. There are two elementary reasons. Firstly, network is regarded as a good tool to deal with a set of interactional systems. Secondly, Lagrange equation is viewed as a classic and representative dynamical model, it can well formulate a class of mechanical systems. More importantly, its dynamics analysis is challenging due to the inherent nonlinearity and strong coupling between its generalized coordinates. As a consequence, there has been an increasing research interests in synchronization and control of networked Lagrange systems [1-3].

Impulsive motion of networked multibody systems arises when the interactional subsystems or agents are characterized by the fact that at certain moments of time they experience abrupt changes of the states due to the following two elementary reasons. The first reason is a directly physical behavior on subsystems, which is mainly caused by impulsive force or impulsive constraint [4, 5]. In recent years, impulsive motion of multibody system has been a significant topic in theoretical research and practical applications. On the other hand, impulsive control strategy based upon impulsive force or impulsive constraint, is viewed as an efficient approach for cooperative control of mechanical systems and recently gained renewed interests.

With the aforementioned background, we in this paper are mainly interested in synchronization motion of networked multibody systems via impulsive cooperative control. The impulsive effects are usually caused by impulsive forces or impulsive constraints. The primary contribution of this paper is to propose a distributed algorithm for the system output synchronization with respect to generalized coordinates and its velocities. We provide the convergence analysis for such algorithms over, respectively, undirected fixed, and then present some generic criteria on exponential convergence for solving synchronization problems.

PROBLEM STATEMENT AND MAIN RESULTS

Consider a team of N agents n-degree networked Lagrange systems:

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i, \ \ i = 1, 2, \cdots, N, \tag{1}$$

where $q_i \in \mathbf{R}^n$ is the vector of generalized configuration coordinates, $\dot{q}_i$ is the vector of generalized velocities, $\ddot{q}_i$ is the vector of generalized acceleration. $M_i(q_i) \in \mathbf{R}^{n\times n}$ is the symmetric positive-definite inertia matrix, $C_i(q_i,\dot{q}_i)\dot{q}_i \in \mathbf{R}^n$ is the vector of Coriolis and centripetal torques, $g_i(q_i) \in \mathbf{R}^n$ is the vector of gravitational torques, $\tau_i \in \mathbf{R}^n$ is the vector of generalized forces acting on the systems.

More generally, we choose a preliminary control input τ_i for each agent i as

$$\tau_i = -M_i(q_i)\lambda\dot{q}_i - C_i(q_i,\dot{q}_i)\lambda q_i + g_i(q_i) + u_i, \ \ i = 1, 2, \cdots, N, \text{ and } u_i = \mu\sum_{k=1}^{\infty}\sum_{j\in\mathbf{N}_i} a_{ij}(r_j - r_i)\delta(t - t_k), \tag{2}$$

Dirac function $\delta(t)$ depicts the impulsive effects of sudden appearances of constraints or abrupt changes in the networked system motion at the time moment $t = t_k$.

Thus, by substituting the control inputs τ_i into system (1), the governing equations of networked multibody systems with impulsive constraints can be given by

$$\begin{cases} \dot{r}_i(t) = -M_i(t)(q_i(t))^{-1}C_i(q_i(t),\dot{q}_i(t))r_i(t), & t \in [t_{k-1}, t_k), \\ \dot{q}_i(t) = r_i(t) - \lambda q_i(t), & t \in [t_{k-1}, t_k), \\ \Delta r_i(t_k) = r_i(t_k) - r_i(t_k^-) = \mu M_i^{-1}(q_i(t)) \sum\limits_{j\in N_i} a_{ij}(r_j(t_k^-) - r_i(t_k^-)), & t = t_k, \\ \Delta q_i(t_k) = q_i(t_k) - q_i(t_k^-) = \mu M_i^{-1}(q_i(t)) \sum\limits_{j\in N_i} a_{ij}(q_j(t_k^-) - q_i(t_k^-)), & t = t_k. \end{cases} \tag{3}$$

[a)] Corresponding author. E-mail: jinzhousu@yahoo.com.cn.

Let $x_i = [r_i, q_i]^{\mathrm{T}}$, $f(x_i) = [-M_i(q_i)^{-1}C_i(q_i, \dot{q}_i)r_i, -\lambda q_i + r_i]^{\mathrm{T}}$, we have the following assumptions A1 and A2:
A1: For any i, there exist two positive constants $K_{\overline{i}},\ K_{\underline{i}}$, such that $M_i^{-1}(q_i) - K_{\overline{i}} I_n \leq 0$, and $M_i^{-1}(q_i) - K_{\underline{i}} I_n \geq 0$.
A2: Function f is one-sided global Lipschitz on $[t_{k-1}, t_k)$, i.e., there exists a constant $\xi > 0$ such that, for all $x_i(t),\ x_j(t) \in \mathbf{R}^{2n}$, $t \in [t_{k-1}, t_k)$, $k \in \mathbf{N}$, $(x_j(t) - x_i(t))^{\mathrm{T}}[f(x_j(t)) - f(x_i(t))] \leq \xi \|x_j(t) - x_i(t)\|^2$.
Let $\overline{K} = \max\{K_{\overline{1}}, K_{\overline{2}}, \cdots, K_{\overline{N}}\}$, $\underline{K} = \min\{K_{\underline{1}}, K_{\underline{2}}, \cdots, K_{\underline{N}}\}$, $\Delta t_k = t_k - t_{k-1}$, the eigenvalues of Laplacian matrix associated to a connected graph G can be ordered as $0 = \lambda_1 < \lambda_2 \leq \cdots \leq \lambda_N$.
Theorem 1 Consider the system described by (1) with the control input τ_i. Assume that the fixed topology graph G is undirected, connected and if $0 < \mu < \frac{2\underline{K}\lambda_2}{\overline{K}^2\lambda_N^2}$ and $\frac{\ln(1-2\mu\underline{K}\lambda_2+\mu^2\overline{K}^2\lambda_N^2)}{2\Delta t_k} + \xi < 0$, then the agents with control constraints (2) will achieve the exponential output synchronization with respect to generalized coordinates q_i and its velocities $\dot{q}_i$.

APPLICATION EXAMPLE AND SIMULATION RESULTS

Consider six two-link revolute manipulators modeled by Lagrange dynamics with fixed topology graph:

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i,\ i = 1, 2, \cdots, 6,\ q_i = [q_{i1}, q_{i2}]^{\mathrm{T}} \in \mathbf{R}^2. \tag{4}$$

The agents are assumed to be communicated using fixed topology graph G as shown in Fig 1. Let $m_1 = 1,\ m_2 = 1.2,\ l_1 = 1.8,\ l_2 = 2$. The initial values can be randomly chosen as $[q_1(0), r_1(0), q_2(0), r_2(0), \cdots, q_6(0), r_6(0)]^{\mathrm{T}} = [2.1881, 2.8324, 2.4257, 2.7911, 2.7724, 2.6510, 2.8351, 2.6248, 2.6079, 2.6178, 3.0589, 2.4992, 2.3171, 2.3684, 2.8079, 2.9579, 2.9801, 2.4665, 2.8703, 2.7024, 2.5252, 2.6350, 2.6155, 2.7761]^{\mathrm{T}}$ in the interval $[2.14, 3.14]$. In simulation, we let $\lambda = 0.9$ and choose $\mu = 0.0486$, $t_k - t_{k-1} = 0.0018$, by Theorem 1, then system (4) is output synchronized, (see Fig. 2), the corresponding generalized coordinates q_i and generalized velocity $\dot{q}_i$ are synchronized.
The condition of Theorem 1 is sufficient but not necessary. Let $\mu = 0.74,\ \Delta t_k = 0.3$, and other parameters are the same as system (4), then outputs synchronization of system (4) is realized (see Fig. 3).

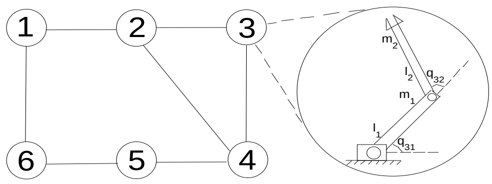

Fig.1: The fixed topology graph of six two-link revolute manipulators (agents).

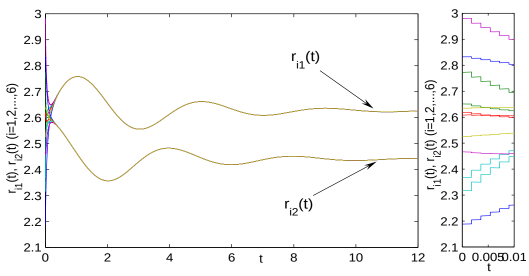

Fig.2: Output $r_i(t)$ synchronization of system (4).

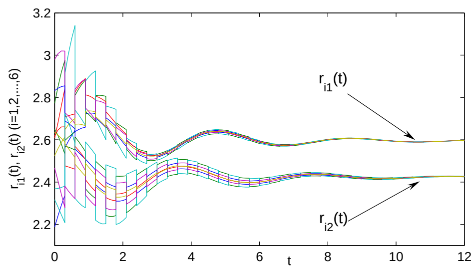

Fig.3: Output $r_i(t)$ synchronization of system (4).

CONCLUSIONS

In this paper, by utilizing cooperative protocol of preliminary feedback control with impulsive constraints, the impulsive synchronization motion of networked multibody systems modeled by Lagrange dynamics with fixed and switching topologies have been investigated. We derive a sufficient condition under which the networked multibody systems can achieve impulsive synchronization motion. To this end, an example and simulations are given to illustrate the analytical results. The analysis procedure developed in this paper can be further generalized to deal with synchronization problems for more general networked multibody systems.

References

[1] Spong, M.W., Chopra, N.: Synchronization of networked Lagrangian systems. Lecture Notes in Control and Information Sciences, Lagrangian and Hamiltonian Methods for Nonlinear Control, Springer-Verlag. **366**, 47-59, 2007.
[2] Nair, A., Leonard, N.E.: Stable synchronization of mechanical system networks. SIAM J. on Contr. and Optim. **47**(2), 661-683, 2008.
[3] Ren, W.: Distributed leaderless consensus algorithms for networked Lagrange systems. Int. J. of Contr. **82**(11), 2137-2149, 2009.
[4] Chang, C.C., Liu, C.Q., et al.: Dynamics of multibody systems subjected to impulsive constraints. Multibody Syst. Dyn. **8**, 161-184, 2002.
[5] Chang, C.C., Peng, S.T.: Impulsive motion of multibody systems. Multibody Syst. Dyn. **17**, 47-70, 2007.