MODELING STICKING, STICK-SLIP AND SLIDING IN MULTI-BODY SYSTEMS WITH MANY FRICTIONAL CONTACTS

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Summary Modeling sticking, stick-slip and sliding often rises the problem of an accurate description of the friction effects at reasonable computation times. This is particularly a problem when there are many frictional contacts and the configurations change at a high rate. The contribution presents an approach to resolve this problem and illustrates it on two examples.

CONFIGURATIONS AND ELASTO-PLASTIC FRICTION MODEL

Equations (1-3) describe the friction force F_f for the three different configurations of friction between two contacting bodies moving relative to each other in tangential direction by velocity $\dot{g}_t$ [2].

$$F_f = \begin{cases} -F_n \mu_{ve}(|\dot{g}_t|) sgn(\dot{g}_t) & \text{if} \quad \dot{g}_t \neq 0 & \text{(Oscillatory) Sliding} & (1) \\ -F_{et} & \text{if} \quad \dot{g}_t = 0, |F_{et}| < F_{ba} & \text{Sticking} & (2) \\ -F_n \mu_{ba} sgn(F_{et}) & \text{if} \quad \dot{g}_t = 0, |F_{et}| \geq F_{ba} & \text{Stick-Slip} & (3) \end{cases}$$

During sliding, the friction coefficient $\mu_{ve} > 0$ is a function of motion velocity and is not defined for $\dot{g}_t = 0$. An example for such a function is the Stribeck-like friction curve in Fig. 1 (E). Oscillatory sliding is captured by Eq. (1) and involves reversion of motion. At the instant of motion reversion it holds $\dot{g}_t = 0$ but no sticking prevails and Eq. (3) applies. During sticking, friction opposes the external force F_{et} applied to the bodies within the plane of contact. At the instant of transition from sticking to sliding (stick-slip), friction is given by the break-away force F_{ba} with $F_{ba} = F_n \mu_{ba}$, $\mu_{ba} > 0$.

Equations (4-7) introduce a force law which is used for multi-body systems [1]. It is capable of modeling the friction configurations in Eq. (1-3). The absolute value of its state μ is the friction coefficient. The switching function $\alpha(\mu)$, Fig. 1 (A), compares μ to the threshold values μ_{ba}, $\mu_{max} > 0$ and determines the rate of change of μ, Eq. (4b). The term *elasto-plastic* comes from modeling the sticking phase by an elastic spring-element which breaks if a certain elongation is exceeded, resulting in a non-elastic, plastic displacement.

$$F_f = -F_n \mu \quad (4a) \quad \dot{\mu} = c\dot{g}_t (1 - \alpha(\mu) \frac{\mu}{\mu_{ve}(|\dot{g}_t|)} sgn(\dot{g}_t)) \quad (4b)$$

$$\alpha(\mu) = \begin{cases} 1 & \text{if} \quad |\mu| > \mu_{ba} & \text{(Oscillatory) Sliding} & (5) \\ 0 & \text{if} \quad |\mu| < \mu_{ba} & \text{Sticking} & (6) \\ \frac{1}{2} sin(\pi \frac{\mu - \frac{1}{2}(\mu_{max} + \mu_{ba})}{\mu_{max} - \mu_{ba}}) + \frac{1}{2} & \text{if} \quad \mu_{ba} \leq |\mu| \leq \mu_{max} & \text{Stick-Slip} & (7) \end{cases}$$

Sticking: Equation (4b) degenerates to $\dot{\mu} = c\dot{g}_t$. Starting from a relaxed state $\mu = 0$ with no friction it holds $\mu = cg_t$. This results in $F_f = -F_n c g_t$, Eq. (4a). Large stiffness c results in a large force F_f opposing motion for any small displacement g_t. The force law behaves like a spring element. By the upper bound μ_{ba} of $|\mu|$,Eq. (6), no distinct dynamics evolves. This results in a quasi-static elongation of the spring by an external force F_{et}.

Stick-Slip: For a maximum allowable displacement g_{ba}, the break-away friction coefficient is determined by $\mu_{ba} = cg_{ba}$. Hence, the break-away displacement g_{ba} results from the physically motivated choice of μ_{ba} and the numerically- and performance-orientated choice of c. For a displacement $g_t > g_{ba}$, a switching process from 0 to 1 occurs. To avoid a step in α and in the switching process in case of $\mu_{ba} \neq \mu_{ve}(|\dot{g}_t| \downarrow 0)$, the values can be set to $\mu_{ba} < \mu_{max} = \mu_{ve}(|\dot{g}_t| \downarrow 0)$. Only for this purpose μ_{max} is introduced; it has no physical meaning like μ_{ba}.

(Oscillatory Sliding): By Eq. (4b) it follows $\dot{\mu} = c\dot{g}_t(1 - \frac{\mu}{\mu_{ve}} sgn(\dot{g}_t))$. This is a control law that adjusts the state μ to the friction coefficient μ_{ve} carrying the sign of the motion direction. The tracking error becomes zero in steady-state, i.e. it holds $\mu = \mu_{ve} sgn(\dot{g}_t)$. Inserting this into Eq. (4a) produces Eq. (1). Increasing c, the convergence to steady-state is faster. However, setting c too high, the transient tracking deteriorates due to overshoots.

Friction in Multi-Body Systems: The equations of motion are given by Eq. (8) with state vector $\boldsymbol{q}$, generalized mass matrix $\boldsymbol{M}(\boldsymbol{q})$ and generalized force vector $\boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}, t)$ comprising the friction force F_f. With the elasto-plastic model, an additional state μ is introduced for each active contact, i.e. it is an additional degree of freedom for friction computation.

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} - \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}, t) = \boldsymbol{0} \quad (8)$$

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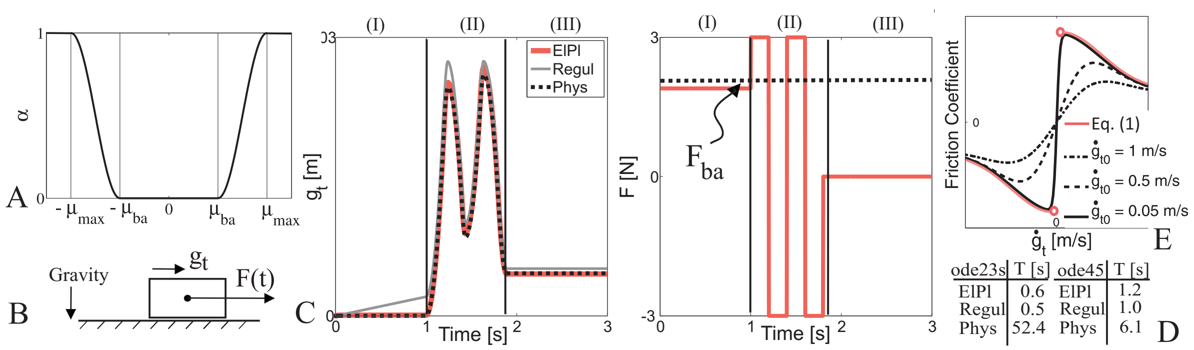
SIMULATIVE EVALUATION OF ELASTO-PLASTIC MODEL

In two examples, the elasto-plastic model (ElPl) is compared to the physical model (Phys) Eq. (1-3) and a regularized approach (Regul). Figure 1 (E) depicts the regularized approach for different regularizations $\dot{g}_{t0}$. A smaller $\dot{g}_{t0}$ better approximates the correct physical curve in Eq. (1) for low velocities but can reduce the integration times, see *Example 2*.

Example 1: For three seconds, a block, Fig. 1 (B), is exposed to a time-dependent force $F(t)$, regarding the block's displacement g_t. In phase (I), Fig. 1 (C), sticking prevails with $F(t)$ being slightly below the break-away force F_{ba}. The physical and elasto-plastic models represent sticking, unlike the regularized approach. In phase (II), a stick-slip transition occurs and siding begins. The elasto-plastic and regularized models follow the physically correct curve by Eq. (1). In phase (III), a slip-stick transition sets in. All models dissipate the kinetic energy. A look at the integration times in Fig. 1 (D) for an explicit (ode45) and an implicit integrator (ode23s) reveals that the regularized approach for this regularization is slightly faster than the elasto-plastic approach, but the elasto-plastic model is more accurate. As the next example shows, the integration time of the regularized approach and its accuracy largely depend on the regularization.



ode23s	T [s]	ode45	T [s]
ElPl	0.6	ElPl	1.2
Regul	0.5	Regul	1.0
Phys	52.4	Phys	6.1

Figure 1. Switching Function (A), Block (B), Friction Configurations (C), Simulation Times (D), Regularized Friction (E).

Example 2: Considered is an AUDI-V6 timing-drive with 250 frictional contacts in the chain strands and additional frictional contacts between chains/sprockets and chains/guides, Fig. 2 (A). Operating at 5700 RPM, high-frequency oscillations occur in the strands. Figure 2 (B) compares the error for different regularizations to the elasto-plastic model. The reference is a regularized model with high accuracy and $\dot{g}_{t0} = 0.01\frac{m}{s}$ which is assumed to precisely represent the physics for sliding, Eq. (1). Displayed are the computation times for a simulated time of 1 second using a Runge-Kutta-Fehlberg integrator of order 2/3. Since the regularized model cannot represent sticking, the comparison is only for oscillatory sliding, showing however the accuracy and computational efficiency of the elasto-plastic model. Figure 2 (C) compares the elasto-plastic model to the regularized model for $\dot{g}_{t0} = 0.01\frac{m}{s}$ for the complete chain frictional power dissipation rate in bank 1.

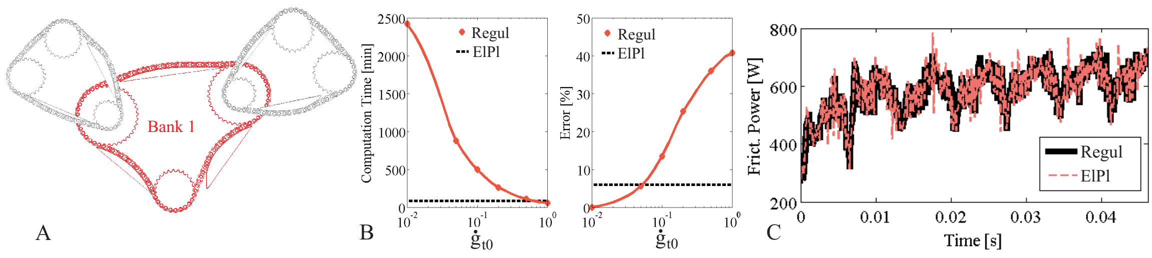


Figure 2. Switching Function (A), Block (B) and Friction Configurations (C).

CONCLUSIONS

The elasto-plastic model is shown to capture the friction phenomena of sticking, stick-slip and sliding. Its computational efficiency, especially for high-frequency oscillations, is pointed out by comparing its integration times to a friction approach with different regularizations. It is predestined for large-scale multi-body systems with many frictional contacts.

References

[1] Dupont P., Hayward V., Armstrong B.: Single State Elasto-Plastic Friction Models. *IEEE Trans. on Autom. Control*, 47 (5): 787-792, 2002
[2] Olsson H., Aström K., Canudas de Wit C., Gäfwert M.: Friction Models and Friction Compensation. *European J. of Control*, 4: 176-195, 1998