ON THE SIMILARITY BETWEEN THE ELECTROMAGNETICS OF MICROSTRIP ANTENNA AND THE MECHANICS OF COMPOSITE SUBSTRATES

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Summary Fiber-reinforced laminated composites in modern aerospace systems, in addition to their superior mechanical properties, offer another advantage of embedding microstrip antennas for communication. An electromagnetic model in spectral domain is developed so that the design of microstrip antenna embedded inside composite laminated substrates are determined by using the vector Hertzian potentials, Galerkin's method, Parseval's theorem in spectral domain, and the antenna dimensions operating at given frequencies are determined by using immittance functions and a suitable set of base functions. The equations governing the electromagnetic wave propagation in spectral domain are in similar matrix forms as the equations governing the mechanics of composite laminated substrates. Similar to the electromagnetic solution, modal solution of a composite laminated substrate can thus be obtained analytically within acceptable accuracy.

INTRODUCTION

Conventional onboard antennas mounted on aircraft fuselage often lead to disturbances in aerodynamic field. In high-performance aircraft applications, where aerodynamics is of major constraints and low profile microstrip antennas may be required. Fiber-reinforced laminated composites in modern aerospace systems, in addition to their superior mechanical properties, offer another advantage of embedding microstrip antennas for communication. The concept of smart antenna module is inspired by the smart structures with built-in sensor/actuator (Yang et al., 2005). For microstrip antennas embedded in composite laminates, the effects of the dielectric layer covering the antennas and the anisotropic permittivity of composite substrates on the antenna performance should be carefully examined. The electromagnetic properties on anisotropic composite substrates may be significantly different from that on common isotropic substrates. Most, if not all of the previous works, however, are applicable only to substrates whose permittivity matrix is isotropic, uniaxial, or biaxial in the context of electromagnetics. Note that in mechanics, uniaxial or biaxial refers to the acting states of normal stress and strain. For microstrip antennas embedded in fiber-reinforced laminated composites, the permittivity matrix has nine distinct dielectric constants. Solution techniques of the spectral domain immittance method and Hertzian vector potentials have been developed (Yang and Hung, 2009) to analyze the antenna performance due to the anisotropic permittivity of composite laminates. Upon close observation, the formulation of the electromagnetic equations is the same form as those governing the modal solution of the mechanics of a composite laminated substrate. Analytical solutions can thus been obtained to facilitate parametric studies of the effects of the substrate's stacking sequence, fiber orientation, and mechanical properties on structural dynamics.

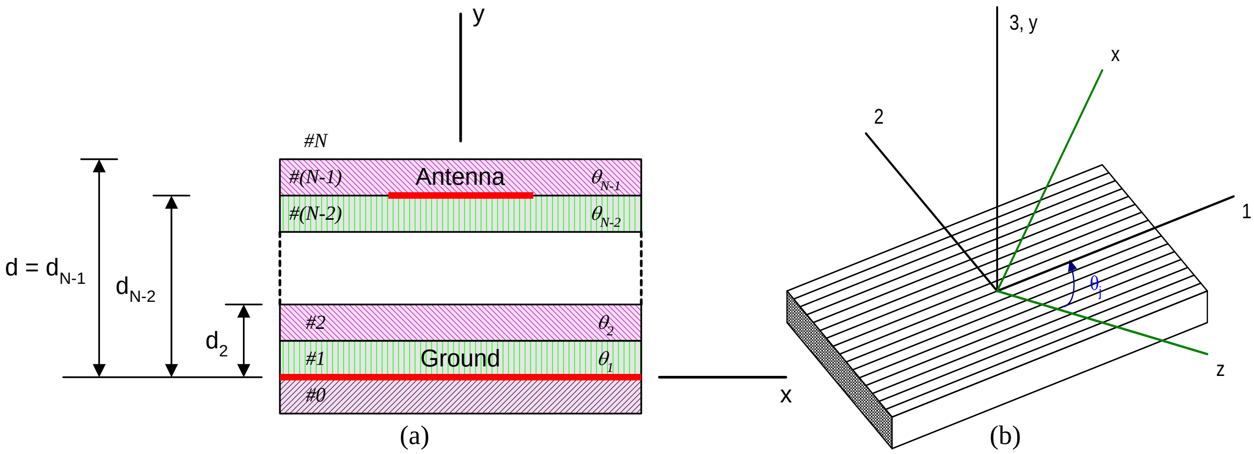

Figure 1 (a) A microstrip antenna embedded inside (N+1) layers of composite lamina of different fiber angle and thickness and (b) illustration of the optical axis of the microstrip antenna and the principal axes of the substrate lamina

ELECTROMAGNECTIC EQUATION

Consider a microstrip antenna embedded inside a composite laminated substrate as in Figure 1(a). The unidirectional composite lamina of the *j*-th ply of a composite laminate as shown in Fig.1(b) has three mutually orthogonal planes of material property symmetry. The direction of the optical axis is along the fiber orientation, each lamina can be taken as a uniaxial substrate if its principal axes, *1-2-3* coordinates, coincide with the space axes, *z-x-y* coordinates because the permittivity tensor is symmetric with respect to the fiber orientation. The permittivity tensor will no longer be diagonal and lead to coupling terms as the fiber orientation changes. The electromagnetic wave propagation can be solved by the electric and magnetic Hertzian potentials for the fiber orientation which will also yield the ordinary waves (TE modes) from the Hertz potential $\boldsymbol{\Pi}_{\boldsymbol{h}_j}$ as

$\boldsymbol{E}_j = -j\omega\mu_0\nabla\times\boldsymbol{\Pi}_{\boldsymbol{h}_j}$, where $\boldsymbol{\Pi}_{\boldsymbol{h}_j} = \Pi_{h_j}\hat{a}_\xi$ is the solution of

$$\nabla^2\boldsymbol{\Pi}_{\boldsymbol{h}_j} + \omega^2\mu_0\varepsilon_{2_j}\boldsymbol{\Pi}_{\boldsymbol{h}_j} = 0 . \tag{1a}$$

j is the imaginary number, ω is the radian frequency of electromagnetic wave propagation. $\mu_0 = 4\pi\times10^{-7}\,H/m$ is the permeability of free-space, $\varepsilon_0 = \frac{1}{36\pi}\times10^{-9}\,F/m$ is the permittivity of free-space, $k_0 = 2\pi/\lambda_0 = 2\pi f\sqrt{\mu_0\varepsilon_0}$ is the wave number or propagation constant of free-space, λ_0 is the wavelength of free-space. Similarly, the extraordinary waves (TM modes) from $\boldsymbol{\Pi}_{\boldsymbol{e}_j}$ are

$\boldsymbol{E}_j = \omega^2\mu_0\varepsilon_0\boldsymbol{\Pi}_{\boldsymbol{e}_j} + \nabla\Psi = \omega^2\mu_0\varepsilon_0\boldsymbol{\Pi}_{\boldsymbol{e}_j} + \frac{\varepsilon_0}{\varepsilon_{2_j}}\nabla\left(\nabla\cdot\boldsymbol{\Pi}_{\boldsymbol{e}_j}\right)$ and $\boldsymbol{H}_j = j\omega\varepsilon_0\nabla\times\boldsymbol{\Pi}_{\boldsymbol{e}_j}$, where $\boldsymbol{\Pi}_{\boldsymbol{e}_j} = \Pi_{e_j}\hat{a}_\xi$ is the solution of

$$\nabla^2\boldsymbol{\Pi}_{\boldsymbol{e}_j} + \omega^2\mu_0\varepsilon_{1_j}\boldsymbol{\Pi}_{\boldsymbol{e}_j} + \left(\frac{\varepsilon_{1_j}-\varepsilon_{2_j}}{\varepsilon_{2_j}}\right)\frac{\partial^2}{\partial\xi^2}\boldsymbol{\Pi}_{\boldsymbol{e}_j} = 0 . \tag{1b}$$

$\boldsymbol{E}(x, y, z, t)$ is the electric field intensity (in V/m or $kg\cdot m/A\cdot\sec^3$), $\boldsymbol{H}(x, y, z, t)$ is the magnetic field intensity (in A/m). Equation (1a and b) can be used to obtain both the field components and the homogeneous wave equation for Hertzian potentials in spectral domain, and the subscript j denotes all the properties of the j-th ply with ply angle ${}_j$.

For each lamina of the composite substrates is not isotropic, the derivation of electromagnetic fields from the Maxwell's equations in source-free region should have the permeability tensor $\boldsymbol{\mu} = \mu_0\boldsymbol{I}$ and the diagonal permittivity tensor. Depending on the orientation, two of the diagonal terms are equal. Let $\hat{a}_\xi$ be the unit vector along the optical axis ξ with a permittivity or dielectric constant ε_1 along the optical axis and ε_2 along the other two axes normal to that axis. The magnetic Hertzian potential $\boldsymbol{\Pi}_{\boldsymbol{h}} = \Pi_h\hat{a}_\xi = \Pi_h\hat{a}_z$ and the electric Hertzian potential is $\boldsymbol{\Pi}_{\boldsymbol{e}} = \Pi_e\hat{a}_\xi = \Pi_e\hat{a}_z$ of the j-th ply lamina in fiber orientation θ_j are $\boldsymbol{\Pi}_{\boldsymbol{h}_j} = \Pi_{h_j}\hat{a}_\xi = \Pi_{h_j}(\cos\theta_j\,\hat{a}_z + \sin\theta_j\,\hat{a}_x)$ and $\boldsymbol{\Pi}_{\mathbf{e}_j} = \Pi_{e_j}\hat{a}_\xi = \Pi_{e_j}(\cos\theta_j\,\hat{a}_z + \sin\theta_j\,\hat{a}_x)$, where k_x and k_z are the Fourier variables and characterized by the wave numbers with respect to x- and z- axis, respectively. When written in terms of Fourier transform pairs of Π_{h_j} and Π_{e_j}, $\tilde{\Pi}_{h_j}$ and $\tilde{\Pi}_{e_j}$ become

$$\frac{\partial^2}{\partial y^2}\tilde{\Pi}_{h_j} - \gamma_{h_j}^2\tilde{\Pi}_{h_j} = 0 \qquad \frac{\partial^2}{\partial y^2}\tilde{\Pi}_{e_j} - \gamma_{e_j}^2\tilde{\Pi}_{e_j} = 0 \tag{2a and b}$$

for the j-th ply, where $\gamma_{h_j}^2 = k_z^2 + k_x^2 - \omega^2\mu_0\varepsilon_{2_j}$, and

$$\gamma_{e_j}^2 = \left(1 + \frac{\varepsilon_{1_j}-\varepsilon_{2_j}}{\varepsilon_{2_j}}\cos^2\theta_j\right)k_z^2 + \left(1 + \frac{\varepsilon_{1_j}-\varepsilon_{2_j}}{\varepsilon_{2_j}}\sin^2\theta_j\right)k_x^2 + 2\frac{\varepsilon_{1_j}-\varepsilon_{2_j}}{\varepsilon_{2_j}}\cos\theta_j\sin\theta_j k_z k_x - \omega^2\mu_0\varepsilon_{1_j}.$$

Equation (2) is the so-called homogeneous wave equations for Hertzian potentials in spectral domain.

MODAL SOULTION

After solving Eq.(2) in spectral domain with boundary conditions, the tangential electric fields on the surface of the top ply can be represented as

$$\begin{Bmatrix}\tilde{E}_x\\ \tilde{E}_z\end{Bmatrix} = \begin{bmatrix}\tilde{Z}_{xx} & \tilde{Z}_{xz}\\ \tilde{Z}_{zx} & \tilde{Z}_{zz}\end{bmatrix}\cdot\begin{Bmatrix}\tilde{J}_x\\ \tilde{J}_z\end{Bmatrix}, \tag{3}$$

where $\tilde{Z}_{ij}$ are the so-called immittance functions in spectral domain. where $\tilde{J}_z$ and $\tilde{J}_x$ are the Fourier transforms of the current densities on microstrip antenna. As the immittance functions being derived from Hertzian potential s and boundary conditions at different interfaces in spectral domain, the unknown spectral current components, $\tilde{J}_z$ and $\tilde{J}_x$, can be expanded in terms of linear combinations of known base functions (Itoh, 1981, Yang and Hung, 2009),

$$\tilde{J}_x(k_z,k_x) = \sum_{m=1}^{M}c_m\tilde{J}_{xm}(k_z,k_x), \quad \tilde{J}_z(k_z,k_x) = \sum_{n=1}^{N}d_n\tilde{J}_{zn}(k_z,k_x), \tag{4}$$

to solve the problem that microstrip antenna on anisotropic substrates. $\tilde{J}_{xm}$ and $\tilde{J}_{zn}$ are the Fourier transforms of the basis functions of currents J_{xm} and J_{zn} which are nonzero only on the antenna, and the numbers of modes for $\tilde{J}_x$ and $\tilde{J}_x$ are M and N, respectively. These two equations will become the homogeneous matrix form as

$$\begin{bmatrix}K_{pm}^{xx} & K_{pn}^{xz}\\ K_{qm}^{zx} & K_{qn}^{zz}\end{bmatrix}\cdot\begin{Bmatrix}c_m\\ d_n\end{Bmatrix} = 0, \tag{5}$$

where the matrix elements are functions of the mechanical and electromagnetical properties of the substrate. Eigensolutions of Eq.(5) determines the microstrip antenna performance.

Table 2 shows design examples of microstrip antenna embedded in 6-ply composite laminate of distinct stacking sequence: symmetric cross-ply, antisymmetric angle-ply, and asymmetric balanced laminates and illustrates the length and width ratios with respect to those of microstrip patch on isotropic substrates. η_2/η_1 is the anisotropic ratio. The dimensions of microstrip patches in indicate that microstrip patches will, in general, no longer be square in composite laminates compared to those on isotropic substrates. However, microstrip antenna embedded in laminae of ply angles $\pm45^\circ$ will remain in square in spite of the anisotropic ratio. Microstrip antenna will not be square when embedded in composite laminates, even if in symmetric stacking. Furthermore, the ply angles of $\pm45^\circ$ explain the so-called Merlin's magic in electromagnetic propagation where good directivity can be achieved.

Table 2 Length and width ratios of rectangular microstrip antenna embedded in 6-ply composite laminate (d = 2.00 cm , without overlay, $\varepsilon_{r2} = 4.70$, f = 2.40 GHz).			
Stacking sequence	Anisotropic ratio (η_2/η_1)	Length ratio (L/L_{iso})	Width ratio (W/W_{iso})
[45/-45/45/45/-45/45]	0.75	1.205	1.205
	0.50	1.100	1.100
[90/90/0/0/90/90]	0.75	1.201	1.205
	0.50	1.112	1.104
[45/-45/-45/45/45/-45]	0.75	1.185	1.185
	0.50	1.148	1.148
[30/45/60/-30/-45/-60]	0.75	1.186	1.182
	0.50	1.127	1.154

SIMILARITY WITH THE MECHANICS EQUATION

Furthermore, Eqs.(3) and (5) are in similar matrix forms as those of composite mechanics (Gibson, 1994). The similarities are listed in Table 1 and it is interesting that electromagnetic wave propagation in spectral domain give the similar matrix forms as mechanical wave in space domain.

Table 1 Similarity of the Mechanical and Electromagnetic Quantities in Composite Materials.			
Mechanical System		Electromagnetic System	
Stress of lamina	σ_i	Electric field of lamina	$\tilde{E}_i$
Strain of lamina	ε_j	Current density of lamina	$\tilde{J}_j$
Transformed lamina stiffness matrix	$\overline{Q}_{ij}$	Immittance function of lamina	$\tilde{Z}_{ij}$
Force per unit length	N_i	Electric field of laminate	$\tilde{E}_{i,N-ply}$
Strain on middle surface	ε_j^0	Current density of laminate	$\tilde{J}_{j,N-ply}$
Laminate extensional stiffness	A_{ij}	Immittance function of laminate	$\tilde{Z}_{ij,N-ply}$
Governing equations for lamina: $\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}=\begin{bmatrix}\overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16}\\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26}\\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66}\end{bmatrix}\cdot\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}$ Governing equations for laminate: $\begin{Bmatrix}N_x\\ N_y\\ N_{xy}\end{Bmatrix}=\begin{bmatrix}A_{11} & A_{12} & A_{16}\\ A_{12} & A_{22} & A_{26}\\ A_{16} & A_{26} & A_{66}\end{bmatrix}\cdot\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}$		Governing equations for lamina: $\begin{Bmatrix}\tilde{E}_x\\ \tilde{E}_z\end{Bmatrix}=\begin{bmatrix}\tilde{Z}_{xx} & \tilde{Z}_{xz}\\ \tilde{Z}_{zx} & \tilde{Z}_{zz}\end{bmatrix}\cdot\begin{Bmatrix}\tilde{J}_x\\ \tilde{J}_z\end{Bmatrix}$ Governing equations for laminate: $\begin{Bmatrix}\tilde{E}_x\\ \tilde{E}_z\end{Bmatrix}_{N-ply}=\begin{bmatrix}\tilde{Z}_{xx} & \tilde{Z}_{xz}\\ \tilde{Z}_{zx} & \tilde{Z}_{zz}\end{bmatrix}_{N-ply}\cdot\begin{Bmatrix}\tilde{J}_x\\ \tilde{J}_z\end{Bmatrix}_{N-ply}$	

SUMMARY AND CONCLUSION

An electromagnetic model in spectral domain is developed so that the design of microstrip antennas are determined by using the vector Hertzian potentials, Galerkin's method, Parseval's theorem in spectral domain, and the antenna dimensions operating at given frequencies are determined by using immittance functions and a suitable set of base functions. Analyses also show that electromagnetic properties of a microstrip antenna on anisotropic substrates are significantly influenced by the anisotropy ratio of dielectric constants and the ply angles. The anisotropy of composite laminates due to different ply angles stacking play an important role in antenna performance.

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