VERTICAL ANGULAR MOMEMTUM MINIMIZATION FOR BIPED ROBOTS WITH KINEMATICALLY REDUNDANT JOINTS

Markus Schwienbacher [a)], Thomas Buschmann & Heinz Ulbrich
Institute of Applied Mechanics, Technische Universität München, 85747 Garching, Germany

Summary We address the problem of angular momentum compensation for kinematically redundant biped robots. This is an important issue in biped walking, since large changes in angular momentum lead to large contact forces which can make the robot slip and fall. In this paper a method to minimize the vertical angular momentum and therefore, reduce the contact slippage is presented. The method is integrated into the inverse kinematics framework and exploits the robot's redundant degrees of freedom. Simulations and experiments with our robot *Lola* are conducted to show the effectiveness of the method. Interestingly this optimization-based formulation generates arm motions similar to those seen in human walking.

INTRODUCTION

Angular momentum reduction/tracking is an active field in biped robotics [1, 2]. A widely used method to control linear and angular momentum of a biped robot is the *resolved momentum control method* proposed by KAJITA ET AL. [3]. However, the method is limited to robots with 6 degree of freedom (DoF) legs and because of the nature of the angular momentum it is unclear how to define an appropriate reference trajectory for the angular momentum in any possible situation. In a recent publication by the authors, a formulation where the set of joints is split up into joints which "produce" (such as legs) and joints which "reduce" (such as arms) angular momentum is described [4]. In this paper a more general optimization-based solution to this problem is presented.

PROBLEM STATEMENT AND OPTIMIZATION

The rotatory slippage of the foot-ground contact depends on a maximum applicable vertical torque due to friction. The slippage torque τ_z is related to the rate of change of the vertical angular momentum $\dot{L}_z \propto \tau_z$. Since $\dot{L}_z$ is acceleration dependent and our inverse kinematics solver works on velocity level, it is a good idea to minimize the vertical angular momentum L_z (and therefore its rate of change).
We define the cost function

$$f = \frac{1}{2} L_z^2 + \Delta H(\boldsymbol{q}) \rightarrow \text{min!} \tag{1}$$

where $\Delta H(\boldsymbol{q})$ is the change of an additional function H of the joint position $\boldsymbol{q} \in \mathbb{R}^n$ during one time step. With the partial derivatives $L_z = (\partial L_z / \partial \dot{\boldsymbol{q}})\, \dot{\boldsymbol{q}} = \boldsymbol{J}_{L,z}\, \dot{\boldsymbol{q}}$ (with the jacobian matrix $\boldsymbol{J}_{L,z} \in \mathbb{R}^{1\times n}$) wrt. joint velocities $\dot{\boldsymbol{q}}$ we have

$$f = \frac{1}{2} \dot{\boldsymbol{q}}^T \boldsymbol{J}_{L,z}^T\, \boldsymbol{J}_{L,z}\, \dot{\boldsymbol{q}} + \alpha_N (\nabla_{\boldsymbol{q}} H)^T \dot{\boldsymbol{q}} \tag{2}$$

where α_N is a gain factor, and $\nabla_{\boldsymbol{q}} H$ the gradient of the H wrt. $\boldsymbol{q}$. Since the matrix $(\boldsymbol{J}_{L,z}^T\, \boldsymbol{J}_{L,z})$ is rank deficient we add an additional term $\delta \boldsymbol{E}$ to obtain a convex problem. The variable δ is a positive constant and $\boldsymbol{E}$ is the unit matrix with appropriate dimensions. This allows us to cast the problem into the *resolved motion rate control method* [5] extended with the framework of redundancy resolution proposed by LIÉGEOIS [6]. The modified optimization problem is

$$f^* = \frac{1}{2} \dot{\boldsymbol{q}}^T \underbrace{(\delta \boldsymbol{E} + \boldsymbol{J}_{L,z}^T\, \boldsymbol{J}_{L,z})}_{\boldsymbol{W} \in \mathbb{R}^{n\times n}} \dot{\boldsymbol{q}} + \alpha_N (\nabla_{\boldsymbol{q}} H)^T \dot{\boldsymbol{q}} \rightarrow \text{min!} \qquad \text{subject to} \quad \dot{\boldsymbol{x}} - \boldsymbol{J}_x\, \dot{\boldsymbol{q}} = \boldsymbol{0} \tag{3}$$

where $\dot{\boldsymbol{x}}$ is the time derivative of the task variables $\boldsymbol{x} \in \mathbb{R}^m$ and $\boldsymbol{J}_x := \partial \boldsymbol{x} / \partial \boldsymbol{q} \in \mathbb{R}^{m\times n}$ is the jacobian matrix of the task. The matrix $\boldsymbol{W}$ is called the *weighting matrix* and must be invertible, hence the additional term $\delta \boldsymbol{E}$.
The solution to the linear quadratic programming problem defined in eq. (3) can be obtained by Lagrange duality to

$$\dot{\boldsymbol{q}} = \boldsymbol{J}_{\boldsymbol{W}}^{\#} \dot{\boldsymbol{x}} - \alpha_N \boldsymbol{N} \boldsymbol{W}^{-1} \boldsymbol{y} \tag{4}$$

with the weighted pseudoinverse $\boldsymbol{J}_{\boldsymbol{W}}^{\#} := \boldsymbol{W}^{-1} \boldsymbol{J}_{\boldsymbol{x}}^T (\boldsymbol{J}_{\boldsymbol{x}} \boldsymbol{W}^{-1} \boldsymbol{J}_{\boldsymbol{x}}^T)^{-1}$, the null-space projection matrix $\boldsymbol{N} := \boldsymbol{E} - \boldsymbol{J}_{\boldsymbol{W}}^{\#} \boldsymbol{J}_{\boldsymbol{x}}$ and the null-space vector $\boldsymbol{y} := (\nabla_{\boldsymbol{q}} H)^T \in \mathbb{R}^m$.

APPLICATION

The proposed scheme of vertical angular momentum minimization has been tested in experiments and in our verified multi-body simulation [7] with our biped robot *Lola*. The robot is 180 cm tall and weighs approximately 60 kg. Fig. 1 shows a photograph and the kinematic configuration of the robot with 25 DoF. The distinguishing characteristics of *Lola* are the redundant kinematic structure with 7-DoF legs, an extremely lightweight construction, and a modular joint design with high power density based on brushless motors. A comprehensive overview of *Lola*'s hardware is given in [8].

[a)] Corresponding author. E-mail: m.schwienbacher@tum.de

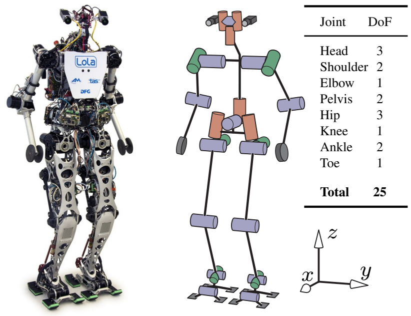

Figure 1. The biped humanoid robot *Lola* with 25 DoFs. Photo and kinematic structure of the system.

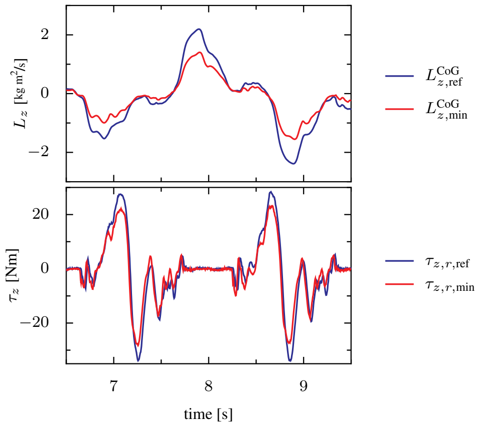

Figure 2. Simulation results comparing the vertical angular momentum about the CoG (above) and the vertical slippage torque acting on the right foot in experiment (below) of a reference solution (blue line; no arm motion) and the proposed method (red line).

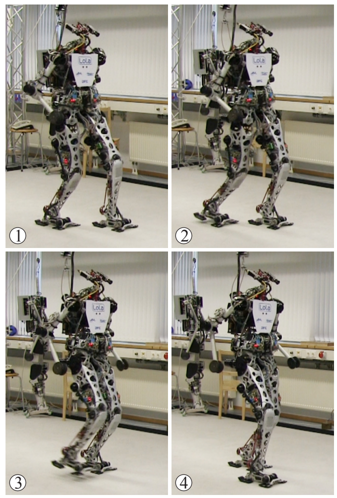

Figure 3. Frame sequence while walking with the proposed method ($\Delta t = 0.24\,\mathrm{s}$ between frames).

Task-Space and Null-Space Definition

The task-space coordinates $\boldsymbol{x}$ are composed of the absolute position of the center of gravity (CoG), the upper body orientation, the positions of the feet relative to the CoG, the orientation of the feet relative to the upper body and the toe joint angles. The desired task-trajectories are output of a planning process described by the authors in [9, 10]. Since the arm movement is not part of the task, they can move freely i.e., stay still or can be used for angular momentum reduction by the algorithm. Therefore, the task-space dimension is smaller than the joint-space dimension and we have a kinematically redundant robot and the null-space vector $\boldsymbol{y}$ influences $\dot{\boldsymbol{q}}$.

The null-space vector consists of several components and is defined as $\boldsymbol{y} = (\partial H_{\mathrm{limit}}/\partial \boldsymbol{q} + \partial H_{\mathrm{cmf}}/\partial \boldsymbol{q} + \partial H_{C} \partial \boldsymbol{q})^T$. Where $\partial H_{\mathrm{limit}}/\partial \boldsymbol{q}$ avoids joint limits, $\partial H_{\mathrm{cmf}}/\partial \boldsymbol{q}$ compensates the drift and keeps the solution close to a comfortable posture and the collision avoidance term $\partial H_C/\partial \boldsymbol{q}$ keeps the body parts separated.

The plots in Fig. 2 compare the resulting vertical angular momentum about the CoG and the slippage torque τ_z for a reference solution (i.e. $\boldsymbol{W} = \boldsymbol{E}$) and the proposed method. Fig. 3 was taken during the experiment using the proposed method (walking parameters: step time: 0.8 s, step length: 0.4 m). It can clearly be seen, that not only the angular momentum is reduced about 40 % but also the slippage torque peaks are lower by approximately 15 %.

CONCLUSIONS

A unified optimization-based method to minimize the vertical angular momentum for kinematically redundant biped robots is presented. Simulations and experiments with the robot *Lola* show the effectiveness of the proposed method.

The drawback of the method is that the calculation of the inverse of the dense weighting matrix $\boldsymbol{W}$ is computational expensive (of $O(n^3)$).

References

[1] A. Goswami and V. Kallem, "Rate of change of angular momentum and balance maintenance of biped robots," in *Proc. IEEE Int. Conf. Robot. Autom. (ICRA)*, 2004.

[2] D. E. Orin and A. Goswami, "Centroidal momentum matrix of a humanoid robot: Structure and properties," in *Proc. IEEE Int. Conf. Intell. Robots Syst. (IROS)*, 2008.

[3] S. Kajita, F. Kanehiro, K. Kaneko, K. Fujiwara, K. Harada, K. Yokoi, and H. Hirukawa, "Resolved Momentum Control: Humanoid Motion Planning based on the Linear and Angular Momentum," in *Proc. IEEE Int. Conf. Intell. Robots Syst. (IROS)*, 2003, pp. 1644–50.

[4] M. Schwienbacher, T. Buschmann, S. Lohmeier, V. Favot, and H. Ulbrich, "Self-Collision Avoidance and Angular Momentum Compensation for a Biped Humanoid Robot," in *Proc. IEEE Int. Conf. Robot. Autom. (ICRA)*, Shanghai, China, May 2011, pp. 581–86.

[5] D. E. Whitney, "Resolved Motion Rate Control of Manipulators and Human Prostheses," *IEEE Trans. Man-Mach. Syst.*, vol. 10, no. 2, pp. 47–53, 1969.

[6] A. Liégeois, "Automatic Supervisory Control of the Configuration and Behavior of Multibody Mechanisms," *IEEE Trans. Syst., Man, Cybern.*, vol. 7, no. 12, pp. 868–71, 1977.

[7] T. Buschmann, S. Lohmeier, H. Ulbrich, and F. Pfeiffer, "Dynamics simulation for a biped robot: modeling and experimental verification," in *Proc. IEEE Int. Conf. Robot. Autom. (ICRA)*, 2006, pp. 2673–8.

[8] S. Lohmeier, T. Buschmann, and H. Ulbrich, "Humanoid robot LOLA," in *Proc. IEEE Int. Conf. Robot. Autom. (ICRA)*, 2009, pp. 775–80.

[9] T. Buschmann, S. Lohmeier, M. Bachmayer, H. Ulbrich, and F. Pfeiffer, "A collocation method for real-time walking pattern generation," in *Proc. IEEE Int. Conf. Humanoid Robots (Humanoids)*, 2007, pp. 1–6.

[10] T. Buschmann, S. Lohmeier, and H. Ulbrich, "Biped walking control based on hybrid position/force control," in *Proc. IEEE Int. Conf. Intell. Robots Syst. (IROS)*, 2009, pp. 3019–24.