PHASE TRANSFORMATIONS LIMIT SURFACES CONSTRUCTION FOR ELASTIC SOLIDS BASED ON EXACT ENERGY LOWER BOUNDS

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Summary Direct and reverse transformations limit surfaces are constructed in strain space for elastic solids undergoing phase transformations. Two approaches are discussed. The first one is based on local thermodynamic equilibrium considerations. We construct the surfaces of the appearance of various new phase equilibrium domains and consider the envelope of such surfaces as a kind of the transformation surface. The second approach is based on exact energy lower bounds. Given new phase volume fraction we construct such a bound using translation estimates. After that, given average strain, we find the equilibrium new phase volume fraction. Then considering the case of vanishing equilibrium volume fraction we construct the transformation limit surface such that not a two-phase equilibrium structure have the energy less than the energy of one-phase state until the limit surface is reached. Finally we compare the transformation surfaces and microstructures obtained by two approaches for the direct and reverse transformations.

TRANSFORMATION SURFACES CONSTRUCTION: TWO APPROACHES

We consider stress-induced phase transformations in elastic solids. The transformation is accompanies by the transformation strain and jumps of elastic moduli. Surface energy is neglected. We focus on direct and reverse transformation surfaces construction in a strain space. Given straining path, the phase transformation cannot start until the external strain attains the transformation surface.

We consider two approaches. The first approach is based on local thermodynamic equilibrium considerations and uses a semi-inverse method. The new phase nucleus shape is prescribed and external strains at which the boundary of such a nucleus can satisfy the local equilibrium conditions (including the Maxwell relation) can be found whereupon geometric parameters of the nucleus shape and the orientation can be found in dependence on the external strains. Then some kind of a transformation surface can be constructed as an envelope of the surfaces which correspond to external strains at which the appearance of different types of new phase nuclei becomes possible. This approach, supplemented by local stability analysis, on one hand allows us to describe specific shapes of new phase nuclei and to find the nucleus geometry in dependence on strain state (layers, ellipsoids, elliptical cylinders – see, e.g., [3] and reference therein), to study local strains in relation with external stains, to estimate stress concentrations due to new phase appearance.

Following this way and summarizing known results for nuclei of various shapes we note that the envelope of transformation surfaces is nonconvex for the direct or reverse transformation in some area of a strain space (see points G in Fig. 1, where the line of equilibrium new phase cylinders appearance crosses with the line of new phase layers appearance), but we do not have now a theorem or postulate according which the limit surface is to be convex as it is in the case of the yield surface in classical plasticity. The major problem of the semi-inverse approach is that the local equilibrium conditions which are used in semi-inverse approach are only necessary conditions for the energy to be minimal. Therefore we are led to inquire whether other two-phase structures can appear before the above mentioned envelopes of the nucleation surfaces is attained. That is why we develop the second approach based on the construction of the exact energy lower bounds of two-phase composites. The transformation limit surface constructed by this approach is such that two-phase structures cannot have the energy less then the energy of one-phase state until the limit surface is reached.

EXACT ENERGY LOWER BOUNDS CONSTRUCTION

First, we consider a two-phase composite at given new phase volume fraction. Basing on translation estimates method (see, e.g., the monographs [2, 4] and reference therein), we construct an exact energy lower bound for such a composite. Note that various particular cases have been already considered, but these studies left open a problem of lower bound construction in case of arbitrary elastic properties of the phases. In the recent paper [1] the cases of cubic symmetry and isotropic phases were presented. The energy lower bounds were constructed and without specifying structure parameters it was proved that the bounds can be attained considering the laminates of the first, second or third rank.

In the present paper we also show that the minimizers can be found basing on considering three types of new phase domains – simple laminates, second and third order laminates but we also show that an additional set of translator parameters and a certain domain of average strains in strain space exists which correspond to the microstructure represented as so called declined second order laminates. Structure parameters are also specified for other average strains. As a result, the exact lower bounds are constructed and the type of the microstructure and microstructure parameters are determined for arbitrary isotropic phases for all average strains. This in turn allows us to go further to construct the transformation limit surface constraction.

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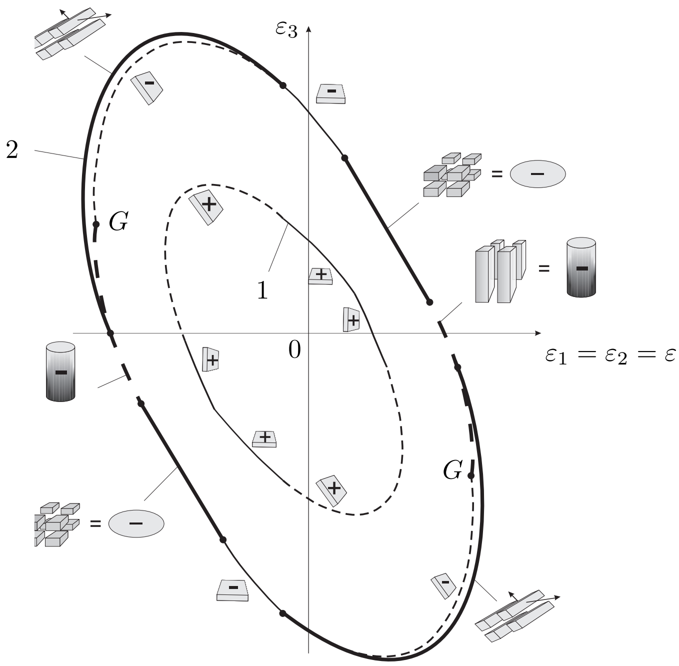

Figure 1. Cross-sections of transformation surfaces by the plane $\varepsilon_1 = \varepsilon_2$.

DIRECT AND REVERSE TRANSFORMATIONS LIMIT SURFACES

We minimize the energy lower bound with respect to the new phase volume fraction and find the equilibrium volume concentration in dependence on average starin. The transformation limit surfaces for the direct and reverse phase transformations are constructed in strain space as a surfaces formed by all strains at which the minimizing new phase equilibrium volume fraction tends to zero.

The example of the transformation surfaces cross-sections is shown in Fig. 1 for the case of sign-definite jump in elasticity tensor. The lines "1" and "2" correspond to the direct and reverse transformation limit surfaces, respectively. Two-phase microstructures which correspond to new phase appearance on the paths of the direct and reverse transformations are different. Only simple laminates (normal or similar to shear band) appear on the path of direct transformation in the presented example. Variety of microstructures is possibleat the reverse transformation. The envelope of the ellipsoids, cylinders and layers nucleation surfaces particulary coincides with the limiting surface. Obviously, the third order laminates are equivalent to ellipsoid, and the second order normal laminates are equivalent to the cylinders. The region of nonconvexity (round points G) is "closed" by the surface that corresponds to the declined second order laminates. Note that the limiting surface is closed if the jump in elasticity tensor is a sign-definite tensor and unclosed in another case.

The results obtained give the complete description of the transformation surface construction for the case of arbitrary isotropic phases and demonstrate how material parameters and straining path affect the values of limit strains and the microstructures.

References

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