A CONSTITUTIVE MODEL FOR COUPLED TRANSFORMATION AND PLASTIC BEHAVIOR OF SHAPE MEMORY ALLOYS

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Summary A three-dimensional phenomenological model for coupled transformation and plastic behavior of shape memory alloys (SMAs) is presented. Two yield functions are introduced and the corresponding evolution rules are obtained respectively for transformation and plastic deformation. Numerical examples shows that it can well describe the main features of SMAs, including the plastic deformation induced obstruction against the inverse transformation, which demonstrates the validity of the proposed model.

INTRODUCTION

In practical application plastic deformation may occur due to stress concentration and for special purposes, and the interaction between plastic deformation and transformation deformation should be considered. Piao[1] speculated that the energy stored during the forward transformation might be relaxed by plastic deformation, and as a result, the inverse transformation temperature, such as A_s and A_f, would be increased. Although some researchers[2-3] investigated the residual irreversible strain during forward martensitic deformation, the modeling involving plastic strain that occurs after transformation and its effect on the following inverse transformation has less been investigated.

A MODEL FOR SMAs TAKING INTO ACCOUNT THE EFFECT OF PLASTIC DEFORMATION

Considering the characteristics of the constitutive behavior of SMAs, we introduce the free energy function as

$$\begin{aligned}\Phi(\boldsymbol{\varepsilon}^e,\boldsymbol{\varepsilon}^t,\boldsymbol{\alpha},p,T)=&\frac{1}{2\rho}\boldsymbol{\varepsilon}^e:\mathbf{C}:\boldsymbol{\varepsilon}^e+\frac{1}{3\rho}C^t\boldsymbol{\varepsilon}^t:\boldsymbol{\varepsilon}^t+\frac{1}{\rho}B\langle T-M_0\rangle\left\|\boldsymbol{\varepsilon}^t\right\|+\frac{1}{\rho}\Gamma(\boldsymbol{\varepsilon}^t)\\&+\frac{1}{3\rho}C^p\boldsymbol{\alpha}:\boldsymbol{\alpha}+\frac{1}{\rho}Y(p)+c_v[(T-T_0)-T\ln(\frac{T}{T_0})]-\frac{1}{\rho}Q(p)\left\|\boldsymbol{\varepsilon}^t\right\|\end{aligned}\tag{1}$$

$\boldsymbol{\varepsilon}^e=\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^t-\boldsymbol{\varepsilon}^p-\boldsymbol{\varepsilon}^\theta$ is elastic strain, and $\boldsymbol{\varepsilon}$, $\boldsymbol{\varepsilon}^t$, $\boldsymbol{\varepsilon}^p$ and $\boldsymbol{\varepsilon}^\theta$ are total strain and its transformation, plastic and thermal components, T is the temperature change, $\boldsymbol{\alpha}$ and p are associated respectively with the kinematic and the isotropic hardening induced by plastic deformation, $\mathbf{C}$ is elasticity tensor, C^t and C^p, are material parameters, ρ and c_v denote respectively the density and specific heat, and are assumed invariant during phase transformation, B represents the sensitivity of the stress with respect to temperature T, and M_0 and T_0 are the reference temperature. $\boldsymbol{\varepsilon}^t$ is subjected to the following constraint at saturated phase transformation

$$\gamma\geq 0,\quad \left\|\boldsymbol{\varepsilon}^t\right\|-\varepsilon_L\leq 0,\quad \Gamma(\boldsymbol{\varepsilon}^t)=\gamma(\left\|\boldsymbol{\varepsilon}^t\right\|-\varepsilon_L)=0\,,\tag{2}$$

in which ε_L is related to saturated phase transformation and γ is a Lagrange multiplier. We define $Q=Q_0[1-\exp(-\xi p)]$ based on the fact that the inverse transformation start temperature A_s depends strongly on irreversible strain [1], where Q_0 and ξ are the material parameters. Using the Clausius–Duhem inequality one obtains

$$\boldsymbol{\sigma}=\rho\frac{\partial\Phi}{\partial\boldsymbol{\varepsilon}^e}=\mathbf{C}:(\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^t-\boldsymbol{\varepsilon}^p-\boldsymbol{\varepsilon}^\theta)\tag{3}$$

and the two corresponding generalized counterparts

$$\boldsymbol{\chi}^t=\rho\frac{\partial\Phi}{\partial\boldsymbol{\varepsilon}^t}=\frac{2}{3}C^t\boldsymbol{\varepsilon}^t+\left(\tau_M(T)-Q(p)+\gamma\right)\frac{\boldsymbol{\varepsilon}^t}{\left\|\boldsymbol{\varepsilon}^t\right\|},\qquad \boldsymbol{\chi}^p=\rho\frac{\partial\Phi}{\partial\boldsymbol{\alpha}}=\frac{2}{3}C^p\boldsymbol{\alpha}\,.\tag{4}$$

$\boldsymbol{\chi}^t$ and $\boldsymbol{\chi}^p$ can be regarded as the transformation- and plastic-back stress. The drag stress of plastic deformation

$$R=\rho\frac{\partial\Phi}{\partial p}=R_0\left[1-\exp(-\zeta p)\right]-Y_0\exp(-\xi p)\left\|\boldsymbol{\varepsilon}^t\right\|\tag{5}$$

where R_0, ζ and Y_0 are material parameters. Introducing the following two dissipation potential functions, Ψ^t and Ψ^p, to describe respectively the evolution of the internal variables of transformation and plastic deformation

$$\Psi^t(\sigma,\boldsymbol{\chi}^t,T)=f^t(\boldsymbol{\sigma},\boldsymbol{\chi}^t,T)^t,\qquad \Psi^p(\sigma,\boldsymbol{\chi}^p,T)=f^p(\boldsymbol{\sigma},\boldsymbol{\chi}^p,T)+\frac{3}{4}\frac{a}{C^p}\boldsymbol{\chi}^p:\boldsymbol{\chi}^p\,,\tag{6}$$

where $f^t(\sigma,\boldsymbol{\chi}^t,T)$ and $f^p(\sigma,\boldsymbol{\chi}^t,T)$ are the yield functions of transformation and plastic deformation and can be expressed as

$$f^t(\sigma,\boldsymbol{\chi}^t,T)=J_2(\boldsymbol{\sigma}'-\boldsymbol{\chi}^t)-\sigma_s^t,\qquad f^p(\boldsymbol{\sigma},\boldsymbol{\chi}^p,\mathrm{R},T)=J_2(\boldsymbol{\sigma}'-\boldsymbol{\chi}^p)-R-\sigma_s^p\,,\tag{7}$$

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a is a constant, $\sigma_s^{\,t}$ and $\sigma_s^{\,p}$ are the yield stresses of transformation and plastic deformation, respectively, and

$$J_2(\boldsymbol{\beta}) = \sqrt{\frac{3}{2}\boldsymbol{\beta}:\boldsymbol{\beta}}\,, \tag{8}$$

where β is an arbitrary tensor β of rank two. The evolution of $\boldsymbol{\varepsilon}^t, \boldsymbol{\varepsilon}^p, \boldsymbol{\alpha}, p$ can be obtained as

$$\dot{\boldsymbol{\varepsilon}}^t = -\dot{\lambda}^t \frac{\partial \Psi^t}{\partial \boldsymbol{\chi}^t} = \dot{\lambda}^t \frac{\partial \Psi^t}{\partial \boldsymbol{\sigma}} = \frac{3}{2}\frac{\boldsymbol{\sigma}' - \boldsymbol{\chi}^{\mathbf{t}}}{J_2(\boldsymbol{\sigma}' - \boldsymbol{\chi}^t)}\dot{\lambda}^t, \qquad \dot{\boldsymbol{\varepsilon}}^p = \dot{\lambda}^p \frac{\partial \Psi^p}{\partial \boldsymbol{\sigma}} = \frac{3}{2}\frac{\boldsymbol{\sigma}' - \boldsymbol{\chi}^p}{J_2(\boldsymbol{\sigma}' - \boldsymbol{\chi}^p)}\dot{\lambda}^p$$

$$\dot{\boldsymbol{\alpha}} = -\dot{\lambda}^p \frac{\partial \Psi^p}{\partial \boldsymbol{\chi}^p} = \frac{3}{2}\frac{\boldsymbol{\sigma}' - \boldsymbol{\chi}^p}{J_2(\boldsymbol{\sigma}' - \boldsymbol{\chi}^p)}\dot{\lambda}^p - \frac{3}{2}\frac{a}{C^p}\boldsymbol{\chi}^p\dot{\lambda}^p \qquad \dot{p} = -\dot{\lambda}^p \frac{\partial \Psi^p}{\partial R} = \dot{\lambda}^p \tag{9}$$

$$\tag{10}$$

where $\dot{\lambda}^p = \sqrt{(2/3)\dot{\boldsymbol{\varepsilon}}^p : \dot{\boldsymbol{\varepsilon}}^p}$, $\dot{\lambda}^t = \sqrt{(2/3)\dot{\boldsymbol{\varepsilon}}^t : \dot{\boldsymbol{\varepsilon}}^t}$.

APPLICATION AND VERIFICATION

Fig. 1 shows the numerical simulation of shape memory effect. The material is stretched at $T < M_f$ to different strain, then unloading, followed by heating. It can be seen that transformation and plastic deformation take place during loading, and residual strain exists after unloading, during heating the transformation strain is recovered and plastic strain remains, The inverse transformation temperature, A_s (marked with small circles), increases with the increase of plastic strain, which is and the tendency is in agreement with the experimental results [1].

Fig. 2 shows the numerical simulation of pseudoelasticity. It can be seen that if unloading occurs before plastic strain appears, the inelastic strain can be recovered. When plastic deformation occurs, it will remain uncovered. With the increase of plastic strain, the stress for inverse transformation is reduced, attributed to that additional energy is needed to compensate the energy for inverse transformation, which is dissipated by plastic deformation. The computed results agree reasonably with the experimental results [4].

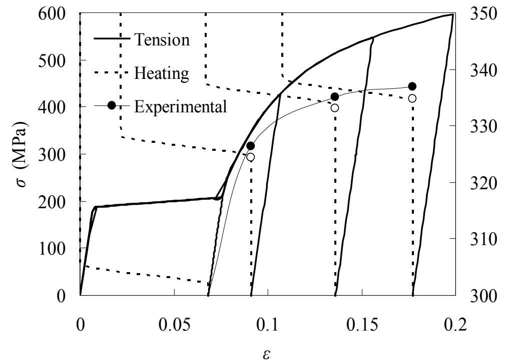

Fig. 1 Simulation of SME and increase of A_s due to plastic deformation

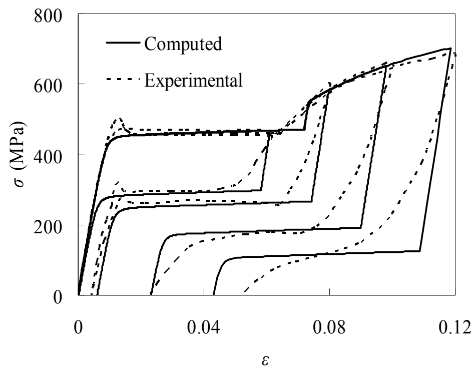

Fig. 2 Simulation of pseudoelasticity and effect of plastic deformation

CONCLUSIONS

A 3-D phenomenological model for SMAs is presented, considering plastic deformation. Making use of a free energy function, the driving force for phase transition and that for plasticity are derived. Two yield functions are introduced to describe the phase transformation and plasticity. The SME and PE of SMAs are analyzed and compared with the experimental results, demonstrating the validity of the proposed model in the description of the main characteristics of SMAs.

ACKNOWLEGEMENT

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References

[1] Piao M., Otsuka K., Miyazaki S., Horikawa H.: Mechanism of the As temperature increase by pre-deformation in thermoelastic alloys. Mater Trans, JIM, 34(10), 919-929. 1993.
[2] Jemal F., Bouraoui T., Zine T. B., Patoor E., Bradaï, C.: Modelling of martensitic transformation and plastic slip effects on the thermo-mechanical behaviour of Fe-based shape memory alloys. Mech Mater 41, 849–856. 2009.
[3] Hartl D. J., Lagoudas D. C.: Constitutive modeling and structural analysis considering simultaneous phase transformation and plastic yield in shape memory alloys. Smart Mater Struct. 18, 104017. 2009.
[4] Wang X., Xu B., Yue Z.: Micromechanical modeling of the effect of plastic deformation on the mechanical behavior in pseudoelastic shape memory alloys. Int. J. Plasticity 24, 1307–1332. 2008.