SHAPE MEMORY ALLOYS AND COLLISIONS

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Summary When a solid made of shape memory alloy (SMA) collides with an obstacle, the velocities of the solid adapt very rapidly to the kinematic constraint due to the obstacle. This phenomenon is very short in time compared to the overall evolution of the structure. Thus we assume it is instantaneous and it results that the velocities are discontinuous with respect to time. Moreover, the temperature is discontinuous due to the dissipative character of the collision. The principle of virtual work and the laws of thermodynamics provide the equation of motion and the constitutive law giving the velocities, the temperature and the composition of the alloy after the collision depending on the velocities, the temperature and the composition before the collision. The simulations agree well with basic mechanical results and, therefore, allow to design structures made of shape memory alloys with the characteristics of dissipating the collision's energy and of coming back to their original shape after collision.

INTRODUCTION

In this paper, we describe the collision of a shape memory alloy, [1], structure with an obstacle, [2]. We assume the collision is instantaneous and velocity $\mathbf{U}(\mathbf{x})$, temperature $T(\mathbf{x})$ and phase volume fractions $\boldsymbol{\beta}(\mathbf{x})$ are discontinuous at collision time, [3, 4]. Vector $\boldsymbol{\beta}(\mathbf{x})$ has components β_1, β_2 and β_3 representing the two martensite phase volume fractions and the austenite one, respectively. We denote:

$$X^+ = \lim_{\Delta t \to 0, \Delta t > 0} X(t+\Delta t), \qquad X^- = \lim_{\Delta t \to 0, \Delta t < 0} X(t+\Delta t), \tag{1}$$

the quantity X after and before the collision. The laws of mechanics provide the equations of motion and the constitutive laws allowing to compute $\mathbf{U}^+(\mathbf{x})$, $T^+(\mathbf{x})$ and $\boldsymbol{\beta}^+(\mathbf{x})$ after the collision, depending on their values before. The predictive theory agree with the physical properties: temperature increases, phase change occurs with the austenite phase appearing when collision is violent and velocity starting the motion after the collision. The resulting equations are easy to solve. The theory which sums up the very sophisticated and fast phenomena occurring during the short duration of the collision has few parameters with a physical meaning. These can be measured and make the theory easy to deal with.

PREDICTIVE THEORY

Consider at time t of collision a solid occupying region Ω with boundary Γ whose outward normal is $\mathbf{n}$ and, for the sake of notation, indicate with $\mathbf{I}$ the identity matrix, with $\underline{T} = (T^+ + T^-)/2$ and with $[X] = X^+ - X^-$. The mass balance gives

$$\textbf{mass balance:} \quad [\beta_1 + \beta_2 + \beta_3] = 0, \tag{2}$$

where the density ρ is assumed to be the same for each phase. The principle of virtual work and the laws of thermodynamics give the three equations of the predictive theory, [3]. Moreover, the constitutive laws are defined by means of suitabe expressions of the free-energy and pseudo-potential of dissipation. Employing an internal-constrained approach, the physical restrictions on the value of the state variables are imposed by means of indicator functions and, thereby, subdifferential sets (denoted by symbol ∂).
Thereby, assuming that the solid is undeformed before collision, the equations for $\mathbf{U}^+$, $\boldsymbol{\beta}^+$ and T^+ result

$$\textbf{equation of motion:} \quad \rho[\mathbf{U}] = \mathrm{div}\boldsymbol{\Sigma} \text{ in } \Omega\,, \tag{3}$$

$$\textbf{phase evolution equation:} \quad c\,[\boldsymbol{\beta}] - \upsilon\Delta\,[\boldsymbol{\beta}] - k\Delta\boldsymbol{\beta}^+ + \partial I_{\mathcal{C}}(\boldsymbol{\beta}^+) + \mathbf{V} = 0 \text{ in } \Omega\,, \tag{4}$$

$$\textbf{energy balance:} \quad (\beta_1^- + \beta_2^- + \beta_3^-)C\,[T] + l_a\,[\beta_3] - \lambda\Delta\underline{T} = \Sigma : D((\mathbf{U}^+ + \mathbf{U}^-)/2) \text{ in } \Omega\,, \tag{5}$$

where symbol : indicates the scalar product, $D(\mathbf{X})$ is the usual strain rate operator, and

$$\boldsymbol{\Sigma} = k_v D(\mathbf{U}^+ + \mathbf{U}^-) + \nu_v \mathrm{div}(\mathbf{U}^+ + \mathbf{U}^-)\mathbf{I}\,, \quad \mathbf{V} = \begin{pmatrix} -P - CT^+\log T^+ \\ -P - CT^+\log T^+ \\ -\frac{l_a}{T_0}(T^+ - T_0) - P - CT^+\log T^+ \end{pmatrix}, \tag{6}$$

$I_{\mathcal{C}}(\boldsymbol{\beta}^+)$ is the indicator function of the set

$$\mathcal{C} = \{(\gamma_1, \gamma_2, \gamma_3) \in \mathbb{R}^3; 0 \le \gamma_i \le 1; \gamma_1 + \gamma_2 + \gamma_3 = 1\}\,, \tag{7}$$

and the percussion pressure P derives from

$$-P \in \partial I_0([\beta_1 + \beta_2 + \beta_3])\,, \tag{8}$$

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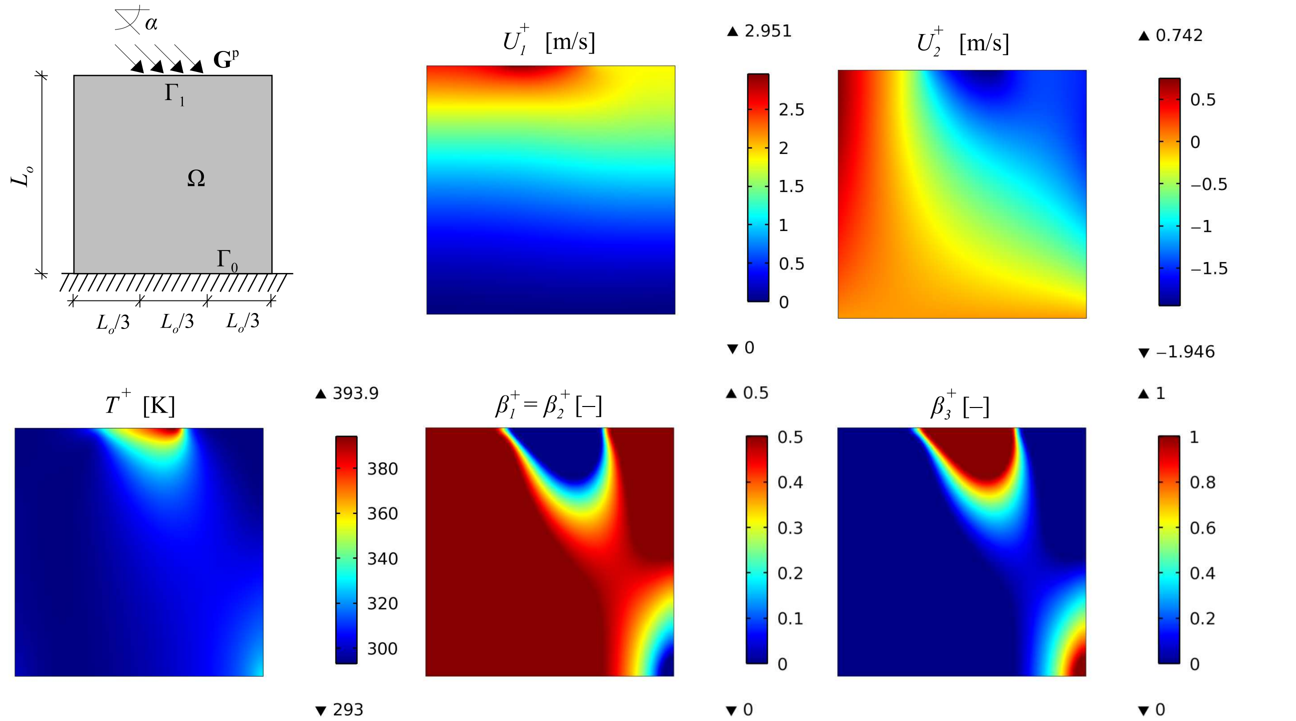

Figure 1. Definition of the simulation and results: velocity $\mathbf{U}(\mathbf{x}) = (U_1(\mathbf{x}), U_2(\mathbf{x}))$, temperature $T(\mathbf{x})$, martensite and austenite volume fractions β_i after collision. Initial conditions: $\mathbf{U}^-(\mathbf{x}) = (0,0)$, $T^-(\mathbf{x}) = 253$ K and $\boldsymbol{\beta}^-(\mathbf{x}) = (0.5, 0.5, 0)$. Parameters' value: $L_o = 2$ m, $\alpha = 60°$, $\|\mathbf{G}^p\| = 2 \cdot 10^5$ N, $\rho = 10^4$ kg/m^3, $k_v = 10^5$ kg/m, $\nu_v = 0.0$ kg/m, $c = 1$ m^2/s^2, $k = 10^{-3}$ m^4/s^2, $\upsilon = 0.0$ m^4/s^2, $\lambda = 10$ W/(mK) , $T_o = 263$ K, $C = 8 \cdot 10^3$ J/(m^{3}K), $l_a = 25$ J/kg.

where I_0 is the indicator function of the origin of $\mathbb{R}$.
The boundary conditions are

$$\boldsymbol{\Sigma}\mathbf{n} = \mathbf{G}^p \text{ on } \Gamma_1\,, \quad \mathbf{U}^+ = \mathbf{U}^- = \mathbf{0} \text{ on } \Gamma_0\,, \quad (k\,\mathrm{grad}\,\boldsymbol{\beta}^+ + \upsilon\,\mathrm{grad}\,[\boldsymbol{\beta}])\mathbf{n} = \mathbf{0} \text{ on } \Gamma\,, \quad \mathrm{grad}\,\underline{T} \cdot \mathbf{n} = 0 \text{ on } \Gamma\,, \tag{9}$$

where percussion $\mathbf{G}^p$, applied by a colliding solid, is given on part Γ_1 of the boundary, the structure is fixed on part Γ_0, no exterior phase-change source and heat impulse are assumed.
Beside the parameters with a classical meaning (heat capacity C, latent heat l_a, thermal conductivity λ), collisions viscosities $k_v \geq 0$, $c \geq 0$ and $\upsilon \geq 0$ appear, as well as a phase interaction parameter $k \geq 0$.

PHYSICAL RESULTS AND CONCLUSIONS

Let us consider a solid (for instance a car) colliding a shape memory alloy solid (for instance a guard-rail) fixed to the ground, assumed to be immobile. For a given percussion, the velocity of the shape memory alloy solid are predicted by the theory in agreement with the physical sense (see Fig. 1). Moreover, temperature increases where the effects of the collision are more significant. Therefore, the theory predicts that austenite appears where the dissipation is more important. This result indicates the great advantage in employing shape memory alloys to dissipate kinetic energy. Moreover, the computed velocity represents the initial one for the post-collision motion and allows to simulate the moving back of the structure close to its original configuration after the collision when cooling.

References

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