A UNIFIED CONSTITUTIVE MODEL FOR INTERFACE DEBONDING AND FRICTION

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Summary Interface debonding and contact interaction are two mechanical phenomena that, despite their many similarities, are often approached from radically different modelling perspectives: the former through constitutive modelling and the latter through enforcement of kinematic constraints. One reason is that, while interface debonding is readily associated with a potential characterizing the internal state of the interface 'material', identifying such a potential is not straightforward for frictional interfaces. This paper presents an interface constitutive model that is capable of coupling damage and plasticity during debonding, as well as accounting for (post-debonding) friction between free surfaces, formulated within a thermodynamically consistent framework.

INTRODUCTION

Unifying the numerical treatment of interface separation and contact has been attempted previously; for instance, within the context of finite elements, 'reverse cohesive contact' is currently used to model delamination [4]. However, this approach calls for the complex and computationally expensive geometric tools that are required to track interacting surfaces in a finite element model. Another approach has been to combine decohesion and friction within a constitutive model, such as in [2], but such models lack the rigor associated with a thermodynamics-based formulation.
In addition to the mixed-mode interaction, any interface debonding model must take into account two key mechanisms of the observed material response in order to have good predictive capabilities: the loss of stiffness (damage) and the permanent deformation (plasticity). For reasons of simplicity, however, most interface constitutive models are pure damage models, equating the total (measured) dissipated energy - which includes the energy lost through friction as well as that released in creating new surfaces - to the fracture toughness Gc.
A coupled interface constitutive model, formulated within a thermodynamic framework, and featuring a strong coupling between damage and friction, highlighted the importance of accounting for the frictional dissipation with dynamic loads such as in the case of impact-induced composite delamination [1]. In the present study, this interface model is improved in order to (1) suppress the *ad hoc* proportional coupling between normal and shear delamination modes, and (2) include post-separation frictional effects.

THERMO-MECHANICAL FORMULATION

Adapting the decomposition of the interface between damaged (cracked) and integral (pristine), proposed by Alfano and Sacco [2], the following expression of the Helmholtz energy potential Ψ is considered:

$$\Psi = \frac{1}{2}(1-D)\,K_n(u_n^e)^2 + \frac{1}{2}D\left[1-H\left(u_n^e\right)\right]K_n(u_n^e)^2 + \frac{1}{2}(1-D)\,K_s(u_{si}^e)^2 + \frac{1}{2}D\left[1-H\left(u_n^e\right)\right]K_s^c(u_sc^e)^2 \tag{1}$$

where D is a scalar variable representing the interface damage; u is the interfacial separation, linked to the elastic (u^e), plastic (u^p) and purely frictional (u^f) parts through the incremental relationships:

$$\delta u_n = \delta u_n^e + \delta u_n^p;\ \ \delta u_s \ = \ \delta u_{si} = \delta u_{si}^e + \delta u_s^p \ = \ \delta u_{sc} = \delta u_{sc}^e + \delta u_s^f$$

K_n and K_s/K_s^c are the initial normal and shear/friction stiffnesses of the interface, respectively, $\langle . \rangle$ are the Macauley brackets, indices n and s stand for normal and shear failure modes and i and c correspond to the integral and cracked part of the interface. The heaviside jump in Equation 1 yield indefinite tractions, which could be treated by a regularization of the Dirac delta function. For simplicity, this development is limited to cases where the compressive normal traction $\langle -t_n \rangle$, if present, is large enough and hence the indefinite terms are neglected in the formula. The normal (t_n) and shear (t_s) tractions, as well as the damage energy (χ) are written as:

$$t_n = \frac{\partial \Psi}{\partial u_n} = \begin{cases} (1-D)\,K_n u_n^e, & u_n^e > 0 \\ K_n u_n^e, & u_n^e \leq 0 \end{cases} \tag{2}$$

$$t_s = \frac{\partial \Psi}{\partial u_s} = \begin{cases} t_{si} = (1-D)\,K_s\left(u_s - u_s^p\right), & u_n^e > 0 \\ t_{si} + t_{sc} = (1-D)\,K_s\left(u_s - u_s^p\right) + DK_s^c\left(u_s - u_s^f\right), & u_n^e \leq 0 \end{cases} \tag{3}$$

$$\chi = -\frac{\partial \Psi}{\partial D} = \chi_n + \chi_{si} - \chi_{sc} \tag{4}$$

From Eq. 2,$sgn\left(t_n\right) = sgn\left(u_n^e\right)$. The damage energies for each mode (normal or shear) are given by:

$$\chi_n = \begin{cases} \frac{t_n^2}{2(1-D)^2 K_n}, & t_n > 0 \\ 0, & t_n \leq 0 \end{cases};\ \chi_{si} = \frac{1}{2}K_s(u_{si}^e)^2 = \frac{t_{si}^2}{2\left(1-D\right)^2 K_s};\ \ \chi_{sc} = \begin{cases} 0, & t_n \leq 0 \\ \frac{t_{sc}^2}{2(1-D)^2 K_s^c}, & t_n > 0 \end{cases} \tag{5}$$

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The choice of $K_s^c \ll K_s$ is consistent with physical observations and ensures that the dissipative force $\chi \geq 0$. The dissipation potential is taken of the quadratic form, Einav et al. [3], $\Phi = \sqrt{\phi_D^2 + \phi_{np}^2 + \phi_{sp}^2 + \phi_{sf}^2}$, where the contributions to the dissipation potential from damage ϕ_D, plasticity in normal (ϕ_{np}) and shear (ϕ_{sp}) and pure shear friction (ϕ_{sf})are respectively given by:

$$\phi_D = \frac{\chi}{\sqrt{\frac{\chi_n cos^2\alpha}{F_n(D)} + \frac{2\chi_{si}(1-D)^2 K_s cos^2\beta}{\left[(1-D)\sqrt{2F_s(D)K_s}+\mu\langle -t_n\rangle\right]^2}}}\delta D; \quad \phi_{sp} = \left[\frac{(1-D)\sqrt{2F_s(D)K_s} + \mu\langle -t_n\rangle}{\sqrt{2K_s\chi_{si}}(1-D)\, sin\beta}\right] t_{si}\delta u_s^p;$$

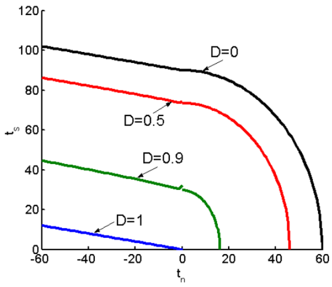
$$\phi_n^p = \frac{\sqrt{F_n(D)}t_n}{sin\alpha\sqrt{\chi_n}}\delta u_n^p \quad \phi_{sf} = \left[\mu\langle -t_n\rangle + (1-D)X\right]\delta u_s^f; \quad X = \frac{t_{sc}}{(1-D)\sqrt{\frac{\chi_{si}}{F_s(D)}\left[1-\left(\frac{t_{s1}^2}{t_s}\right)^2\right]}} \tag{6}$$

Here, the parameters $cos^2\alpha$ and $cos^2\beta$, that are straightforwardly measured experimentally [1], provide the coupling between damage and plasticity for the normal and shear mode, respectively; μ is a friction coefficient related to the surface roughness; X is a function of the tractions and damage that ensures the continuity between tensile and compressive regime; $F_n(D)$ and $F_s(D)$ are damage evolution functions for the normal mode and shear mode, respectively (If linear softening is assumed, the expressions for $F_n(D)$ and $F_s(D)$ are given in [1]). The dissipation terms can then be directly substituted in the following to yield the evolution rules for the internal variables:

$$\delta D = 2\delta\lambda\frac{\chi}{\left(\frac{\partial\phi_D}{\partial\delta D}\right)^2}; \quad \delta u_n^p = 2\delta\lambda\frac{t_n}{\left(\frac{\partial\phi_{np}}{\partial\delta u_n^p}\right)^2}; \quad \delta u_s^p = 2\delta\lambda\frac{t_{si}}{\left(\frac{\partial\phi_{sp}}{\partial\delta u_s^p}\right)^2}; \quad \delta u_s^f = 2\delta\lambda\frac{t_{sc}}{\left(\frac{\partial\phi_{sf}}{\partial\delta u_s^f}\right)^2} \tag{7}$$

FAILURE SURFACE

The corresponding failure surface in stress space is obtained as per Equation 8 and illustrated in Figure 1. A very slight



$$y = \frac{\frac{t_n^2}{2(1-D)^2K_n}}{F_n(D)} + \left[\frac{t_{sc}}{\mu\langle -t_n\rangle + (1-D)X}\right]^2 + \frac{\frac{t_{si}^2}{2(1-D)^2K_s}}{F_s(D)\left[1+\frac{\mu\langle -t_n\rangle}{\sqrt{2(1-D)^2F_s(D)K_s}}\right]^2} - 1 = 0 \tag{8}$$

Figure 1. Loading curve for the unified decohesion/friction model

discontinuity can be seen the surface along the $t_n = 0$ axis, for damage values $0 < D < 1$; however, phenomenologically, this model captures well the classical response of cohesive and frictional interfaces. In particular, it is straightforward to see that for free surfaces($D = 1$) under compression ($t_n < 0$), the Mohr-Coulomb friction criterion is recovered: $t_s^2 = \mu^2 t_n^2$. In tension ($t_n > 0$), the mixed-mode interaction described in [1] is fully recovered.

CONCLUSION

The interface constitutive model introduced here is capable to handle mode mixity, coupling between damage and plasticity in cohesive debonding and finally friction between free surfaces. The model's parameters - $cos^2\alpha$, $cos^2\beta$ and μ - and functions governing the failure process are fully identifiable experimentally [1]. This model is currently implemented in an in-house analysis code based on the Material Point Method (MPM). The MPM with its trivial contact detection, combined with the ability of the present model to reproduce the salient features of both debonding and interaction of surfaces, will yield a computationally efficient technique to simulate dynamic fragmentation.

References

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