A HIGHER ORDER SOLUTION FOR CYLINDRICAL BENDING VIBRATIONS OF FUNCTIONALLY GRADED PLATES

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Summary A higher order solution is presented for cylindrical bending vibrations of functionally graded (FG) plates. Solution for two-constituent metal-ceramic functionally graded plates bound by power-law is presented in this paper. Through-the-thickness variations of the volume fractions of the constituent materials are used and effective material properties at a point are estimated by the Mori-Tanaka scheme. Natural frequencies, displacements and stresses are evaluated and compared with exact, classical plate, and third-order shear deformation theories for FG plates

INTRODUCTION

Due to high specific strength and stiffness to weight ratio, composite materials are having their wide applications in space structures. Laminates of such composite materials are bonded together to achieve superior stiffness and strength. However, because of immediate change in the material properties at interfaces between the laminates, large interlaminar stresses develop leading to delamination. To avoid such failure, 'functionally graded materials' (FGM) are used. Elastic properties in FGM are continuous in a particular direction, especially in thickness direction of plate structure. This can be achieved by changing gradually the volume fraction of the constituent materials.
In this paper, a higher order shear and normal deformation theory (HOSNT8) is used to investigate free vibrations of FGM plates under cylindrical bending. Functionally graded (FG) material is modelled by Mori-Tanaka [1] power law scheme, which is exactly implemented without any simplifications. Natural frequencies, displacements and stresses are calculated and compared with Third Order Shear Deformation Theory (TSDT), Classical Plate Theory (CPT) and exact solution [2].

HIGHER ORDER FORMULATION IN FREE VIBRATIONS

A functionally graded (FG) plate whose one dimension, y, is infinitely long and is simply supported along edges parallel to *y*-axis is shown in Fig. 1. Dimension of FG plate along x-axis is *a* and thickness *h* along *z*-axis. Mid-plane of FG plate is at +*h*/2 and - *h*/2 from top and bottom surfaces respectively. The FG plate is approximated as plane strain problem under cylindrical bending.

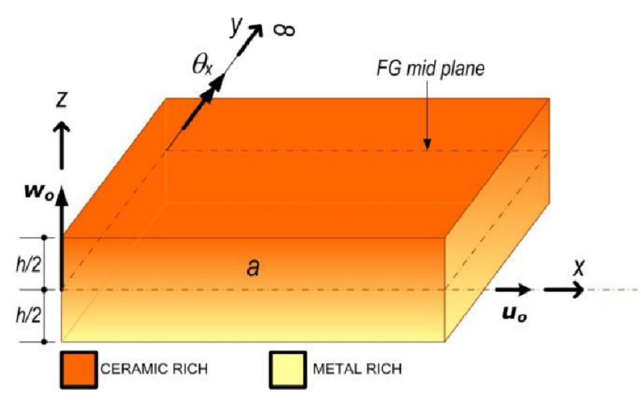

Fig. 1 Geometry of FG plate under cylindrical bending with positive displacement components

Displacement field

In order to approximate the 2D elasticity problem to a 1D plate problem, the displacement components $u(x,z,t)$ and $w(x, z,t)$ at any point in the plate space are expanded in a Taylor series in terms of thickness coordinate *z* as,
Model HOSNT8:

$$u(x,z,t) = u_0(x,t) + z\theta_x(x,t) + z^2u_0^*(x,t) + z^3\theta_x^*(x,t)$$
$$w(x,z,t) = w_0(x,t) + z\theta_z(x,t) + z^2w_0^*(x,t) + z^3\theta_z^*(x,t) \quad (1)$$

The parameter u_0 is in-plane displacement and w_0 is the transverse displacement at the middle plane where, θ_x is the rotation of the normal to the middle-plane about *y*-axis. The other parameters u_0^*, w_0^*, θ_x^*, θ_z^* are the corresponding

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higher order terms in the Taylor's series expansion defined at mid-plane. Navier's solution is adopted to suit the boundary conditions for two opposite edges simply supported parallel to y-axis.

MATERIAL MODELING

A functionally graded composite material is fabricated by blending two discrete materials like a metal and a ceramic. Effective elastic moduli of graded composites can be evaluated based on their volume fraction distribution and dispersed phase. Several micromechanics models have been developed to calculate effective properties of macroscopically homogeneous composite materials and one of them is Mori–Tanaka scheme [1]. Elastic properties of the plate are assumed to vary through the thickness of the plate with a variation of the volume fractions of the two materials in between top and bottom of the FG plate. The modulus of elasticity E and mass density ρ are assumed to be in terms of a simple power law distribution and Poisson's ratio ν is assumed to be constant [3].

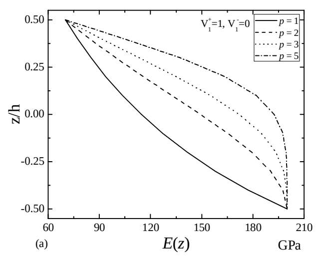

Fig. 2 Variation of Young's modulus across the thickness with Mori-Tanaka scheme

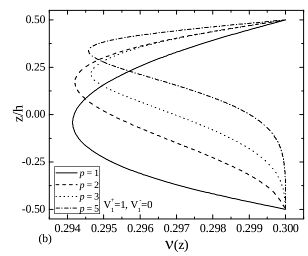

Fig. 3 Variation of Poisson's ratio across the thickness with Mori-Tanaka scheme

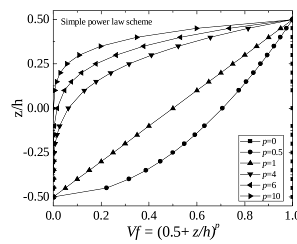

Fig. 4 Variation of volume fraction across the thickness with simple power law

Young's modulus and Poisson's ratio are plotted across the thickness in Figs. 2 and 3 as per Mori-Tanaka scheme. Volume fraction variation across the thickness in plotted according to simple power law scheme in Fig. 4.

RESULTS

The constituent materials of the functionally graded plate are Aluminum and Zirconia with material properties as follows Al: E_m = 70 GPa, ν_m = 0.3, ρ_m = 2702 Kg/m3, ZrO_2: E_c = 200 GPa, ν_c = 0.3, ρ_m = 5700 Kg/m^3. Non-Dimensional flexure mode natural frequency, in-plane displacement and normal stress are plotted in Figs. 5, 6 and 7 respectively.

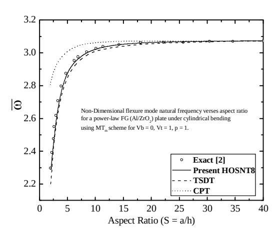

Fig. 5 Non-Dimensional flexure mode natural frequency verses aspect ratio

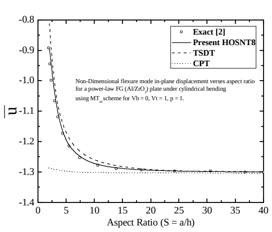

Fig. 6 Non-Dimensional flexure mode in-plane displacement verses aspect ratio

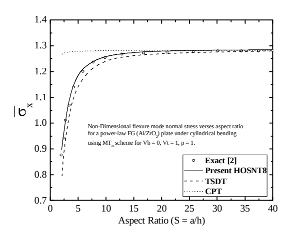

Fig. 7 Non-Dimensional flexure mode normal stress verses aspect ratio

CONCLUSIONS

Exact Mori-Tanaka blending scheme is used to model the FG material in thickness direction of plate. Present HOSNT8 model shows excellent agreement with exact solutions for natural frequencies, displacements and stresses when compared with third order shear deformation theory (TSDT) and classical plate theory (CPT).

References

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