MICROMECHANICS-BASED MODELING OF POROUS MATERIALS FOR APPLICATION TO ELASTO-PLASTIC INDENTATION ANALYSIS

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Summary In this contribution, the hardness properties of porous materials, consisting of an elastoplastic cohesive-frictional matrix phase and a pore phase, are investigated. For this purpose, effective elastic, plastic, and hardening properties are derived with micromechanical methods, taking the post-yield behavior of the material into account. The so-obtained material model is implemented in a numerical tool, providing access to the material hardness as a function of underlying material composition and morphology.

MOTIVATION

Material hardness is commonly assessed by indentation experiments. It is affected by the material properties as well as by the shape of the penetrating body (indenter). Research developments nowadays allow to deduce certain material properties (stiffness, strength, and time-depended behavior) from hardness measurements. While first approaches of identification of material properties were based on the assumption of monolithic materials, Cariou et al. [1] applied limit analyis for accessing the packing density of porous materials from indentation experiments. Since the material is locally subjected to large deformation in course of indentation, the post-yield behavior of materials, which is disregarded in limit analysis, and its effect on the material strength is the focus of the present contribution. Based on a micromechanics-based model, scaling functions relating material properties (properties of the matrix phase, porosity) to the material hardness are presented while considering two modes of post-yield behavior for the matrix phase, i.e., brittle and ductile.

METHOLOGY

The problem is treated at two length scales, i.e., at the macroscale where the indentation experiment is simulated and at the microscale where the properties of an equivalent homogeneous material (EHM) are derived. At the microscale, the matrix phase is supposed to exhibit linear-elastic behavior within the elastic domain of stress states defined by the Drucker-Prager criterion:

$$f_M(\underline{\underline{\sigma}}) = \sigma_d + \alpha\sigma_m - c \leq 0, \quad \sigma_d = \sqrt{\frac{1}{2}\underline{\underline{s}} : \underline{\underline{s}}}, \quad \sigma_m = \frac{1}{3}\mathrm{tr}(\underline{\underline{\sigma}}) \,, \tag{1}$$

with $\underline{\underline{s}}$ being the deviatoric part of the stress tensor $\underline{\underline{\sigma}}$ and α, and c material parameters. Application of the nonlinear homogenization scheme presented in [2] and using the Mori-Tanaka homogenization scheme yields the macroscopic yield criterion of the EHM [3]:

$$f_{hom}(\underline{\underline{\Sigma}}, \varphi) = \left(\frac{3}{4}\varphi - \alpha^2\right)\Sigma_m^2 + 2\alpha c(1-\varphi)\Sigma_m + \left(\frac{2}{3}\varphi + 1\right)\Sigma_d^2 - c^2(1-\varphi)^2 \leq 0 \,, \tag{2}$$

with the deviatoric and hydrostatic stress, Σ_d and Σ_m, being the macroscopic equivalents to σ_d and σ_m, and φ the volume fraction of the pore phase. The elastic behavior of the EHM is obtained by the Mori-Tanaka scheme. For modeling of the post-yield behavior, two micromechanics-based models are proposed: one for a highly ductile and one for a brittle matrix phase. In both models, volume preserving yielding of the matrix phase is supposed. In case of a highly ductile matrix phase (ideal plasticity), φ in Equation (2) is regarded as the current porosity and, thus, as hardening variable. According to [3], the plastic potential - on which the flow rule is based - is given by

$$g_{hom}(\underline{\underline{\Sigma}}, \varphi) = \frac{3}{4}\varphi\Sigma_m^2 + \left(\frac{2}{3}\varphi + 1\right)\Sigma_d^2 \,. \tag{3}$$

For brittle materials, on the other hand, the onset of macroscopic yielding is associated with local damage and softening of the matrix phase. Thus, this local mode of compaction (decrease of φ) does not lead to hardening. At the macroscopic scale, this behavior appears to be ideal plastic in compression, requiring that φ in Equation (2) must remain constant. Accordingly, φ in Equation (2) is replaced by the initial porosity for this type of matrix material. With the assumption that the stress state remains unchanged after onset of yielding for a constant deformation rate, the plastic potential reads

$$g_{hom}(\underline{\underline{\Sigma}}) = \underline{\underline{\Sigma}} : \mathbb{D} : \underline{\underline{\Sigma}} \,, \tag{4}$$

where $\mathbb{D}$ is the inverse of the macroscopic elasticity tensor. Only after full compaction, characterized by $\varphi = 0$, the material strength is assumed to increase.

At the macroscale, the indentation experiment is simulated by means of the finite-element method, exploiting the axisymmetry of the problem. By applying the Π-theorem according to [4], the numerically-obtained results are expressed in a dimensionless format.

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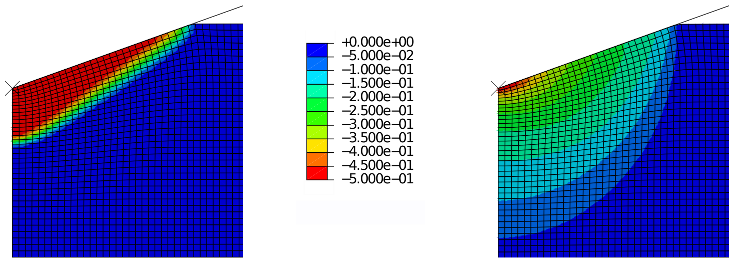
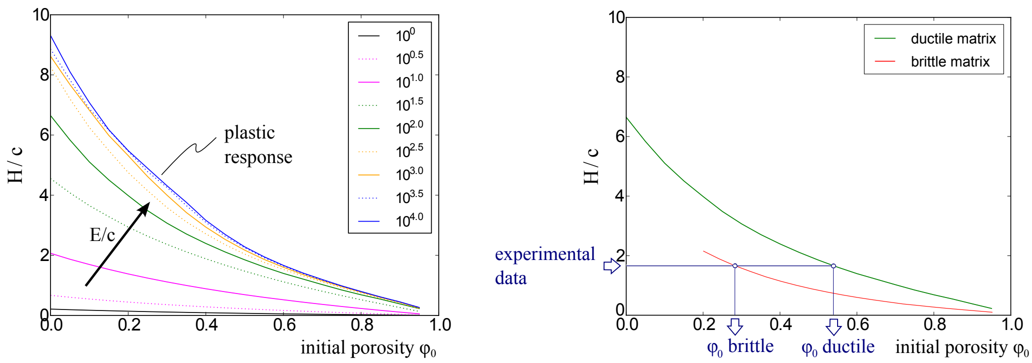


(a) brittle matrix (b) ductile matrix

Figure 1: Numerically-obtained distribution of the change of porosity $\varphi - \varphi_0$ ($\varphi_0 = 0.5$).



(a) scaling functions ($\alpha = 0.25$, ductile matrix material, Berkovich indenter) for different E/c ratios. (b) parameter identification for known values of α and E/c.

Figure 2: Typical scaling functions relating the dimensionless hardness to the initial porosity φ_0

RESULTS

The change of porosity during the indentation process is illustrated in Figure 1. For brittle matrix materials, the fully compacted material is concentrated right underneath the indenter while it is more gradually distributed for ductile matrix materials. As regards the latter, any decrease of porosity is associated with hardening. Thus, the spatial distribution of porosity change and, consequently, hardening leads to a stiffer indentation response as compared to the case of a brittle matrix material.

The hardness, expressed in dimensionless format as H/c, is the force related to the projected area of imprint. In Figure 2(a), H/c is exemplarily shown for a certain friction coefficient, mode of ductility, and indenter geometry.

Simulations also give insight into the deformation behavior of the material surface during indentation (sink-in/pile-up effect). Hence, the indentation depth can be linked with the current area in contact. For small E/c ratios (dominant elastic material response), there is a sink-in which is largely independent of the initial porosity. The pile-up effect only occurs for dense materials ($\varphi < 0.35$), with $E/c > 10^2$.

CONCLUSIONS

The scaling functions obtained from the proposed micromechnics-based model for porous material allow us to identify material properties from indentation tests, considering ductile and brittle matrix material as illustrated in Figure 2. The two given scaling functions given in Figure 2(b) highlight the influence of ductility of the matrix phase (in comparison to brittle matrix behaviour), significantly affecting the material hardness. Thus, an adequate post-yield constitutive model - as proposed in this paper - is essential for reliable modeling of the indentation process of porous materials.

References

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