ANALYSIS OF EFFECTIVE PROPERTIES OF COMPOSITES BY THE BOUNDARY ELEMENT METHOD

Piotr Fedeliński[*], Radosław Górski[*], Grzegorz Dziatkiewicz[*] & Jacek Ptaszny[*]
[*]*Department of Strength of Materials and Computational Mechanics, Silesian University of Technology, 44-100 Gliwice, Konarskiego 18A, Poland*

Summary In this work, different formulations of the boundary element method (BEM) are presented for an analysis of composites containing rigid or deformable fibres and inclusions. The developed computer codes are used to compute effective material properties by considering a representative volume element (RVE) or a unit cell. The results computed by the proposed BEM are compared with available experimental, analytical or numerical results, shown in the literature.

INTRODUCTION

The effective properties of composite materials are required during the design of structures. The overall properties can be determined by applying analytical, empirical or numerical methods. Among most popular computer methods of modelling composites are the finite element method (FEM) and the boundary element method (BEM). The BEM [1] is particularly suitable for modelling composites because of its high accuracy for materials in complex stress state and easy modification of geometry. Effective material properties can be computed numerically by considering a unit cell or a representative volume element (RVE) of a composite [2].
Several authors performed the static analysis of linear-elastic materials with inclusions, which modeled RVEs or unit cells of non-homogeneous materials, by using different formulations of the BEM. Yao et al. [3] applied the formulation for many identical inclusions to the evaluation of elastic constants of composites with direct bonding between inclusions and matrix, and also with interphases between inclusions and matrix. Liu et al. [4] applied the fast multipole BEM to analyze composites reinforced by nanotubes. They calculated effective material properties of composites modelled as three-dimensional structures having large number of degrees of freedom. Qin [5] considered the example of a piezoelectric square RVE with a circular rigid insulating fibre analyzed by the self-consistent – BEM and Mori-Tanaka – BEM approaches to determine the effective properties.

MODELLING OF COMPOSITES BY THE BOUNDARY ELEMENT METHOD

Composites can be modelled using different approaches of the boundary element method. Three cases are considered in the present work - composites containing: rigid stiffeners, deformable stiffeners and deformable inclusions. Stiffeners are attached to the matrix and inclusions are embedded in the matrix. The matrix in each case is modelled using the boundary element method. The rigid stiffeners are modelled as rigid bodies which move with the deformed matrix. The deformable stiffeners are modelled using beam finite elements [6]. The deformable inclusions are modelled using boundary elements [7]. It is assumed that the matrix and reinforcement are made of homogenous and linear-elastic materials. The inclusions can be isotropic or anisotropic. The external and internal boundaries of the matrix are divided into boundary elements. Displacements and tractions within each boundary element are interpolated using nodal values and shape functions. The boundary integral equations are used for nodes along external and internal boundaries of the matrix. The interaction forces between the rigid or deformable stiffener and the matrix are treated as particular body forces distributed only along the internal boundary. Therefore the formulations require integration along the external and internal boundaries. Along the interface between the matrix and the reinforcement the conditions of compatibility of displacements and equilibrium of tractions should be satisfied. The present formulations are extended to piezoelectric composites by using generalized quantities, i.e. generalized displacements and tractions including electrical degrees of freedom [8]. The proposed methods allow direct computing of unknown displacements and tractions along the external boundaries and along interfaces between the reinforcement and matrix.
The boundary element formulations are used to model different composites. Polymer/clay nanocomposites are modelled using deformable stiffeners and deformable inclusions, epoxy-glass composites and piezoelectric composites are modelled using deformable inclusions. Representative volume elements are applied to compute effective material properties of the polymer/clay nanocomposites and epoxy-glass composites, and the unit cell method is used for the piezoelectric composites. The influence of volume fraction of inclusions on effective properties is investigated.
The analysis of polymer/clay nanocomposites demonstrates that modelling nanoparticles as deformable stiffeners or their stacks as deformable isotropic or anisotropic inclusions gives similar effective Young's moduli of the nanocomposite. The effective Young's moduli for polymer/clay nanocomposites and epoxy-glass composites agree well with results obtained by experimental, analytical and other numerical results presented in the literature.
One of the considered numerical examples – a composite with rigid fibres is shown below. Other numerical examples will be presented during the conference and in a full-length paper.

NUMERICAL EXAMPLE – COMPOSITE WITH RIGID FIBRES

A representative volume element of length $2b$ and height $2h$ contains 13 rigid fibres of length $2l$, as shown in Fig. 1a. The horizontal distance between centres of neighbouring fibres is d_1 and the vertical distance is d_2. The ratios of dimensions are: b/l=5, h/l=4, d_1/l=3 and d_2/l=1.6. The material of the plate has the Poisson ratio ν=0.3 and is in plane strain state. The plate is subjected to the horizontal loading q_1. Each rigid fibre is divided into 8 boundary elements and the external boundary into 72 boundary elements. The results for a single fibre in an infinite plate are compared with analytical results [9], showing very good agreement. The stress distribution σ_{11}/q_1 in the matrix, in the marked field in Fig. 1a, is shown in Fig. 1b. The relative effective Young modulus for the considered distribution of fibres is E_c/E_m=1.345, where E_c and E_m are the Young modulus of the composite and the matrix, respectively. The effective Poisson ratio of the composite is the same as for the matrix. The influence of the horizontal distance between the fibres on the effective Young modulus is analyzed. The dimensions of fibres are constant and the dimensions of the plate are changed proportionally to the distance between the fibres. The dependence of the relative Young modulus on the horizontal distance is presented in Fig. 1c. If the horizontal distance is smaller than half of the length of the fibre the stiffness of the composite significantly increases. For the increasing distance between the fibres the stiffness of the composite tends to the stiffness of the matrix.

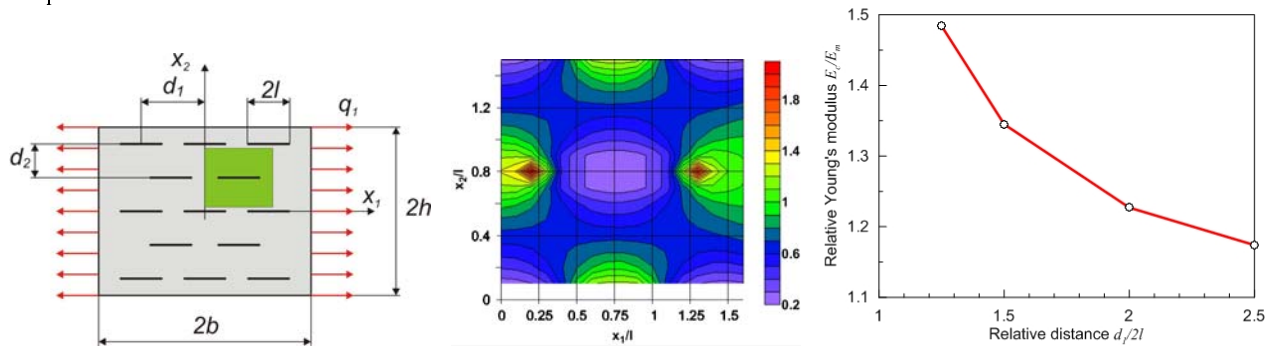

Figure 1: Rigid fibres in a composite: a) dimensions and loading, b) stress distributions in the matrix σ_{11}/q_1, c) influence of the distance between the fibres on the relative effective Young modulus E_c/E_m

CONCLUSIONS

The boundary element method can be used to analyze efficiently composites containing rigid and deformable fibres and inclusions. The BEM allows accurate computation of stresses in composites and easy modification of the reinforcements. The proposed method can be applied to analyze effective properties of epoxy-glass composites, nanocomposites and piezocomposites.

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