THREE-DIMENSIONAL PIEZOELASTICITY SOLUTION FOR HYBRID PIEZOELECTRIC LAMINATED PLATES FEATURING VISCOELASTIC INTERFACES

A. Kumar[**], S. Kapuria[*a)], N. K. Gupta[*]
[*]*Department of Applied Mechanics, Indian Institute of Technology, Delhi, 110016, India*
[**]*Virtaul Reality Lab, Central Mechnical Engineering Research Institute, Durgapur, 713209, India*

Summary The time dependent response of simply supported piezoelectric laminated rectangular plates with viscoelastic interfaces under static loading is investigated. The governing equations of three-dimensional piezoelasticity with Kelvin-Voigt interface model are solved exactly for the spatial variations of the field variables and using time power series expansion for their time variation. The effect of viscoelastic interfaces on the electromechanical response and on sensing and actuation authority of the piezo-layers is studied.

INTRODUCTION

Interfacial bonding imperfections commonly get introduced in laminated structures during manufacturing and service, affecting their integrity. For smart laminates with piezoelectric sensors/actuators, it also affects the sensing and actuation ability of the piezoelectric layers. The most commonly used interface model to analyze the behavior of imperfect interfaces is the spring layer model [1]. However, at high temperatures, the interfacial bonding exhibits viscoelastic characteristics. Sometimes, viscous interfaces may also be artificially introduced to enhance passive damping. This results in a time dependent response under static loading. He and Jiang [2] studied the transient mechanical response of elastic composites with viscous interfaces. It was revealed that viscous interfaces would loose the ability to transfer shear stresses as time reaches infinity, which must be estimated and accounted for in the design. According to Fan and Wang [3], the viscoelastic model is more appropriate to characterize interlaminar adhesion under high temperature condition. Yan et al. [4] presented an analytical solution of elastic laminated plates with viscoelastic interfaces using Kelvin-Voigt interface model and the time series expansion technique [5] for the time variation. This work presents a three dimensional piezoelasticity solution for the time dependent response of piezoelectric laminated simply-supported rectangular plates with viscoelastic interfaces to study the its influence on the electromechanical response and sensing/actuation authority of the sensor/actuator layers.

PROBLEM FORMULATION AND SOLUTION

Consider a hybrid laminated rectangular plate (Fig. 1) made of L orthotropic layers. Some of the layers are of piezoelectric material of class mm2 symmetry (e.g. PZT, PVDF) with poling along the z axis. The plate is simply supported and electrically grounded at the edges:

$$\begin{aligned} u = v = \sigma_x = \phi = 0 \ \text{ at } x = 0, a \\ u = v = \sigma_y = \phi = 0 \ \text{ at } y = 0, b \end{aligned} \tag{1}$$

where u, v and w are the displacements, σ_i, τ_{ij} are the normal and shear stresses, and ϕ denotes electric potential. The plate is subjected to pressure p_1 and p_2 at the bottom and top surfaces, respectively, where one may prescribe electric potential ϕ_i for actuation or transverse electric displacement D_i for sensing:

$$\begin{aligned} \tau_{zx} = \tau_{zy} = 0, \sigma_z = p_1, \phi = \phi_1 \text{ or } D_z = D_1 \text{ at bottom} \\ \tau_{zx} = \tau_{zy} = 0, \sigma_z = p_2, \phi = \phi_2 \text{ or } D_z = D_2 \text{ at top} \end{aligned} \tag{2}$$

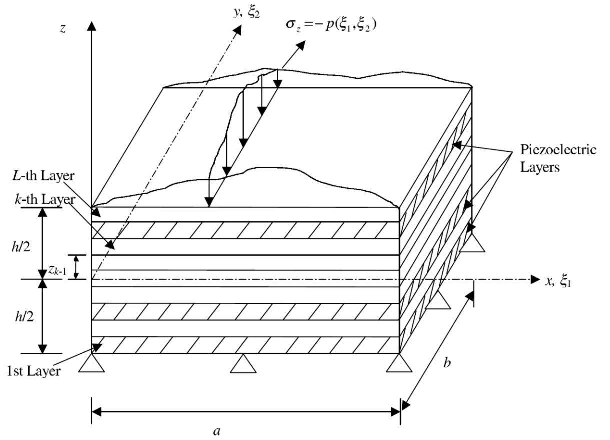


Figure 1. Geometry of the simply supported hybrid plate

The relative sliding of layers at a thin viscoelastic interface between kth and $(k+1)$th layers, and the equilibrium of forces across it can be represented by the following conditions, $\delta_x^{(k)}$ and $\delta_y^{(k)}$ being the relative displacements at the interface:

$$(\tau_{zx}, \tau_{yz}, \sigma_z, D_z)^{(k)} = (\tau_{zx}, \tau_{yz}, \sigma_z, D_z)^{(k+1)}, (w, \phi)^{(k+1)} = (w, \phi)^{(k)}, u^{(k+1)} = u^{(k)} + \delta_x^{(k)}, v^{(k+1)} = v^{(k)} + \delta_y^{(k)} \tag{3}$$

These and the corresponding transverse shear stresses at an interface obey the Kelvin-Voigt relation, i.e., $\tau_{zs}^{(k)} = K_s^{(k)}\delta_s^{(k)} + C_s^{(k)}\dot{\delta}^{(k)}$, $(s = x, y)$, where $K_s^{(k)}$ and $C_s^{(k)}$ are elastic and viscous coefficients for kth interface. The response entities of each layer along with the prescribed electromechanical loading functions are expanded in double Fourier series to satisfy the boundary conditions at simply supported ends. The piezoelasticity constitutive, and stress and charge equilibrium equations for the piezoelectric media [1] are transformed into eight first order ordinary differential equations (ODEs) in thickness coordinate with constant coefficients of the form $\frac{dX}{dz} = QX$, where $X = [u\, v\, w\, \tau_{zx}\, \tau_{yz}\, \sigma_z\, \phi D_z]^T$. The general closed form solution of the ODEs is obtained as $X = \sum_{j=1}^{8} F_j(z)C_j$. C_j are arbitrary constants to be obtained from boundary and interface relations. To model the time dependent interface conditions, the power series expansion technique proposed by Chen and Lee [5] is adopted, wherein the time domain is divided into a series of equal

a) Corresponding author. E-mail: kapuria@am.iitd.ac.in

intervals of magnitude Δt. For $(p+1)^{\text{th}}$ time interval $[p\Delta t, (p+1)\Delta t](p = 0, 1, 2, 3...)$, it is assumed that

$$\chi = \chi_{p,0} + (t - p\Delta t)\chi_{p,1} + (t - p\Delta t)^2\chi_{p,2} + (t - p\Delta t)^3\chi_{p,3} + ... \tag{4}$$

$$\mathrm{d}\chi/\mathrm{d}t = \chi_{p,1} + 2(t - p\Delta t)\chi_{p,2} + 3(t - p\Delta t)^2\chi_{p,3} + ... \tag{5}$$

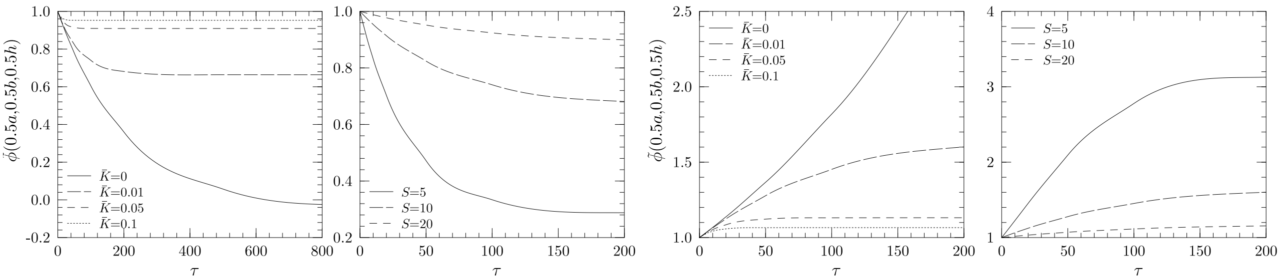
where χ may be any of u, v, w, τ_{zx}, τ_{yz}, σ_z, ϕ, D_z, δ_x and δ_y. $\chi_{p,i}$ are the time-independent constants. $\delta_{s_{0,0}} = 0$ from the zero initial condition ($\delta = 0$ at $t = 0$). The remaining elements of $\chi_{0,0}$ can now be obtained from closed form solution for X using relations obtained by substituting the expansion (4) into the boundary and interface conditions (2) and (3) and equating the coefficients of same order of $(t - p\Delta t)$. The subsequent $\delta_{s_{0,i}}$, $(i = 1, 2, ...)$ can be calculated from the following recurrence relation obtained by substituting the expansions (4) and (5) into Kelvin-Voigt relation and equating the coefficients of the same order of $(t - p\Delta t)$:

$$\delta_{s_{p,i}} = \frac{\tau_{zs_{p,i-1}} - K_s\delta_{s_{p,i-1}}}{iC_s}; s = x, y \tag{6}$$

Thus all the constants $\chi_{0,i}$ can be obtained. Now by setting $t = \Delta t$ in (4) all the physical variables $\chi_{1,0}$ including $\delta_{s_{1,0}}$ can be computed. The same procedure is repeated for $p = 0, 1, 2, \ldots$. The final solution at any instant depends on the chosen value of Δt and number of total terms adopted in series expansion in Eq. (4).

NUMERICAL RESULTS AND DISCUSSION

To show the effect of viscoelastic interfacial sliding between elastic substrate and the sensor/actuator layer on the sensing/actuation authority of the piezoelectric layer, a simply supported square hybrid plate (thickness parameter $S = a/h$) made of elastic substrate of four layers of graphite-epoxy (Gr/Ep) laminate ($0°/90°/90°/0°$) with a PZT-5A layer of thickness $0.1h$ bonded on top, is considered. The plies of the substrate are of equal thickness. The stiffness and viscous coefficients of the viscoelastic interface are same in x and y directions and are nondimensionalized as $\bar{K} = Kh/c_0$ and $\bar{C} = C/C_0$. The time variable is nondimensionalized as $\tau = c_0t/(hC_0)$. We choose $c_0 = c_{11}$, $C_0 = C$ where c_{11} is the stiffness coefficient of Gr/Ep. Unless otherwise specified, $S = 10, \bar{K} = 0.01$. A bisinusoidal pressure load $p_2 = -p_0 \sin(\pi x/a)\sin(\pi y/b)$ is applied on the top of the plate keeping $D_2 = 0$ and $\phi_1 = 0$. For sensing authority, sensory potential $\tilde{\phi}$ induced under this load at time τ, normalized with respect to that developed at $\tau = 0$, is plotted in Fig. 2(a), for different values of dimensionless interface stiffness coefficient $\bar{K}$ and thickness parameter S. The sensory potential decreases with time, the rate of decrease being higher for thicker plates and those with low interface stiffness.



(a) sensory potential (b) actuation potential

Figure 2. Effect of viscoelastic interface on sensing and actuation authority of piezoelectric layer

For the actuation authority, the potential load $\phi_2 = \phi_0 \sin(\pi x/a)\sin(\pi y/b)$ required to be applied (with $\phi_1 = 0$) to reduce the central deflection caused by the pressure load to half its value, normalized with respect to that required at $\tau = 0$ (i.e. for perfect bonding), is plotted in Fig. 2(b). The actuation potential required to be applied for the same percent reduction in deflection increases with time, the rate of increase being higher for thicker plates and those with low interface stiffness.

CONCLUSIONS

The viscoelastic interfacial imperfection has significant effect on the stress distributions, displacements as well as sensing and actuation authority of sensor and actuator layers in smart laminated structures. The exact 3D solution can be used as benchmark for assessing other 2D laminate theories modeling viscoelastic imperfections.

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