CHIRAL EFFECT IN PLANAR ISOTROPIC MICROPOLAR ELASTICITY

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Summary The existing non-centrosymmetric micropolar theory fails to model the chiral effect inherent in two-dimensional isotropic chiral solids, we therefore propose a new micropolar constitutive relation with an additional material parameter characterizing the coupling between rotation and bulk deformation. Homogenization of a triangular chiral lattice provides analytically the effective constants in the theory. The unique chiral behaviour of the lattice is also demonstrated by numerical examples.

INTRODUCTION

An object is said to be chiral, or with handedness, if it cannot be superposed to its mirror image. A chiral material should be described by an adequate constitutive equation with handedness. Classical Cauchy elasticity[1] cannot admit chirality, hence the micropolar theory is usually adopted[2]. A general isotropic chiral (also known as non-centrosymmetric) micropolar theory introduces three additional material constants, which change their signs according to the handedness of the microstructure to represent the effect of chirality[1]. However, when this theory is applied to a planar case, the resulting theory becomes basically non-chiral[3]. Therefore the basic characteristic of a planar chiral solid cannot be properly modeled by the existing theory. The objective of the paper is to propose a continuum model to capture the chiral effect for planar isotropic chiral solids. The proposed theory is also applied to a planar triangular chiral lattice to demonstrate its capacity.

PLANAR ISOTROPIC MICROPOLAR MODEL WITH CHIRALITY

Characterization of chiral effect is closely related to the concept of pseudo (or axial) tensors and ordinary (or polar) tensors. Both types of tensors coexist in continuum mechanics, but they must be combined to keep the strain energy density independent of handedness, i.e. an ordinary scalar. The general micropolar constitutive equations read

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} + B_{ijkl}\phi_{k,l}\,, \qquad m_{ij} = B_{ijkl}\varepsilon_{kl} + D_{ijkl}\phi_{k,l}\,, \tag{1}$$

where σ and m are the stress and couple stress tensors, ε and ϕ are the displacement and micro-rotation, respectively. For an isotropic solid, the material tensors C, D and B take the following general form

$$B_{ijkl} = B_1\delta_{ij}\delta_{kl} + B_2\delta_{ik}\delta_{jl} + B_3\delta_{il}\delta_{jk}\,. \tag{2}$$

A micropolar solid with $B \neq 0$ is usually referred as non-centrosymmetric. A planar micropolar problem in $x_1 - x_2$ plane is defined by $u_3 = \phi_1 = \phi_2 = \partial/\partial x_3 = 0$, while the non zero quantities are u_α, ϕ_3, $\phi_{3,\alpha}$, $\varepsilon_{\alpha\beta}$, $\sigma_{\alpha\beta}$ and $m_{\alpha 3}$, respectively, with Greek subscripts ranging from 1 to 2. It is easy to verify that when reduced to a 2D case the chirality represented by the isotropic B tensor disappears. However in the 2D case, the Levi-Civita tensor e_{ijk} is restricted to the form $e_{ijk} \equiv e_{3\alpha\beta}$, which is in-plane isotropic and equivalent to an isotropic tensor of rank two. Together with the Kronecker delta $\delta_{\alpha\beta}$, we have more choices to construct a 2D isotropic tensor of rank four, besides the basic form, Eq.(2). Representing the involved elastic tensors in the extended form, and re-examining the resulting energy density, a new isotropic chiral constitutive equation is derived for planar chiral solids.

$$\sigma_{\alpha\beta} = \lambda\delta_{\alpha\beta}\varepsilon_{\rho\rho} + (\mu+\kappa)\varepsilon_{\alpha\beta} + (\mu-\kappa)\varepsilon_{\beta\alpha} + A\delta_{\alpha\beta}e_{3\gamma\rho}\varepsilon_{\gamma\rho} + Ae_{3\alpha\beta}\varepsilon_{\rho\rho}\,, \tag{3a}$$

$$m_{\alpha 3} = \gamma\phi_{3,\alpha}\,, \tag{3b}$$

where λ, μ, κ and γ are the traditional micropolar constants. A new material constant A is introduced to characterize the charality. Its absolute value is bounded by the requirement of thermal dynamic stability, i.e., $A^2 < (\lambda+\mu)\kappa$. This material constant characterizes the coupling between rotation and bulk deformation (Fig.1). With the proposed constitutive equation, the governing equations of the proposed theory read

$$\rho\ddot{u} = (\lambda+2\mu)u_{,xx} + (\mu+\kappa)u_{,yy} + (\lambda+\mu-\kappa)v_{,xy} + 2\kappa\phi_{,y} - A(v_{,xx} - v_{,yy} - 2u_{,xy} - 2\phi_{,x}) \tag{4a}$$

$$\rho\ddot{v} = (\mu+\kappa)v_{,xx} + (\lambda+2\mu)v_{,yy} + (\lambda+\mu-\kappa)u_{,xy} - 2\kappa\phi_{,x} - A(u_{,xx} - u_{,yy} + 2v_{,xy} - 2\phi_{,y}) \tag{4b}$$

$$j\ddot{\phi} = \gamma(\phi_{,xx} + \phi_{,yy}) - 4\kappa\phi + 2\kappa(v_{,x} - u_{,y}) - 2A(u_{,x} + v_{,y}) \tag{4c}$$

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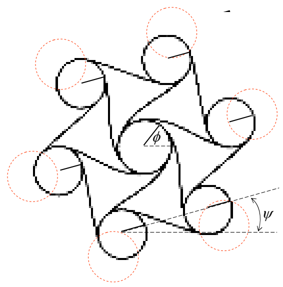

Fig.1 the chiral effect in the planar micropolar medium

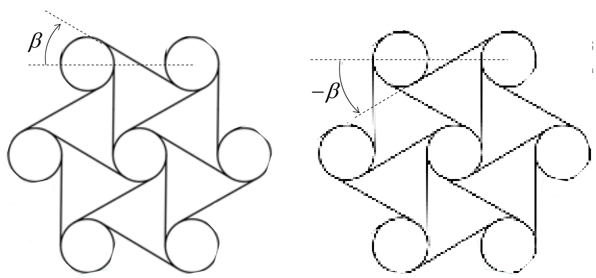

Fig.2 Chiral lattice with (a) β >0 and its handedness reversed pattern with (b) β <0.

APPLICATION ON 2D TRIANGULAR CHIRAL LATTICE

In the framework of the proposed chiral theory, a well known isotropic 2D structure, the triangular chiral lattice[4] is homogenized to analytically give all the effective material constants in Eq.(3). The lattice is originally proposed to produce a material with negative Poisson's ratio. A topology parameter β not only affects the microstructural pattern but also determine the handedness of the material by its sign, as shown in Fig.2. The circles of the lattice are assumed to be rigid and for the deformable ligaments the Euler-Bernoulli beam model is adopted. We show here only the effective chiral parameter

$$A = \frac{\sqrt{3}E_s}{2}\eta\left(\eta^2 - \cos^2\beta\right)\sec\beta\tan\beta \tag{5}$$

where E_s is the Young's modulus of the lattice material and η is the slenderness ratio of the ligaments. It is seen that the chiral parameter is an odd function of β. When the lattice is flipped over (handedness reversed), A flips its sign while the other effective parameters remain the same. To further demonstrate the capacity of the proposed theory, a one-dimensional static tension problem is solved at first. The result (Fig.3) shows that even a simple tension can induce rotation and lateral displacement fields inside of the material, hence create an inclined deformed sample. This behaviour is confirmed by the accurate discrete model. The non-chiral micropolar or Cauchy model always gives zero lateral displacement and rotation field. For a plane wave loading, there is no longer pure longitudinal and transverse waves, and all the wave modes are of mixed types(polarization angle is other than 0 and 90 degree, shown in Fig.4) and dispersive. Mixed wave modes with isotropic frequency dispersion are also observed (Fig.5), which is a fundamental property of a planar isotropic chiral solid. This special wave phenomenon is not reported before. These results are also confirmed by the exact solution of the corresponding discrete model.

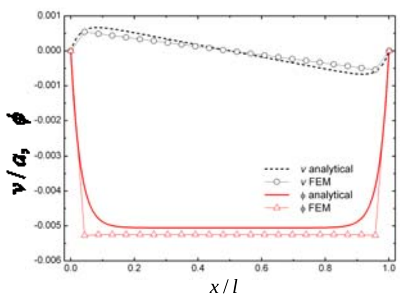

Fig.3 lateral displacement and microrotaiton fields of 1D tension

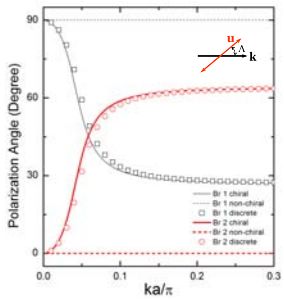

Fig.4 Polarization angle for the first two branches

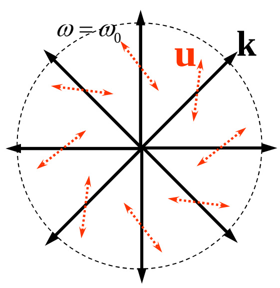

Fig.5 Schematic wave behavior of the 2D chiral media

CONCLUSIONS

We propose in this work a continuum theory to capture the chiral effect of 2D chiral solids, as well as a homogenization technique to analytically derive the effective material constants for a triangular chiral lattice. A number of new behaviours are predicted by the proposed theory and are confirmed by the corresponding discrete model. The proposed method provides a useful tool to investigate the chiral effect on mechanical behaviour for planar isotropic chiral solids.

References

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