MODELLING OF ANISOTROPIC COMPOSITES BY NEWLY DEVELOPED HFS-FEM

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Summary In this work anisotropic composite is modeled efficiently and accurately by a newly proposed hybrid finite element method based on Green's functions. The hybrid method is formulated based on two independent assumptions: intra-element field covering the element domain and inter-element frame field along the element boundary. A modified functional, which satisfies the governing equation, boundary and continuity conditions between elements, is proposed to derive the element stiffness. In this work, the foundational solutions of anisotropic materials derived from the elegant and powerful Stroh formalism are employed to approximate the intra-element displacement field of the elements, while the polynomial shape functions used in traditional FEM are utilized to interpolate the frame field. Numerical results show that the proposed method is accurate and efficient in modeling anisotropic composite, and can be easily extended to analyze composite laminates with good accuracy.

INTRODUCTION

In materials science, composite laminates are usually assemblies of layers of fibrous composite materials which can be joined to provide required engineering properties, such as in-plane stiffness, bending stiffness, strength, and coefficient of thermal expansion [1]. In the viewpoint of micromechanics, the fiber and matrix of each lamina can be treated as the inclusion and matrix, respectively, while from the viewpoint of macromechanics, each lamina can be treated as anisotropic or orthotropic materials and the laminates can be dealt as a general anisotropic body by classical lamination theory. Hence, the analysis of anisotropic bodies is important for understanding of the micro- or macro-mechanical behavior of composites [1]. Stroh formalism [2, 3], which has been shown to be elegant and powerful, is widely used to find the analytical solutions for anisotropic problems [2]. However, numerical methods such as FEM and BEM are usually resorted to solve more complex problems with complicated boundary constraints and loading conditions. In this paper a new hybrid finite element formulation (HFS-FEM) for anisotropic composite has been developed based on the associated fundamental solutions derived from Stroh formalism. As an alternative to the Hybrid Trefftz-FEM, this new method inherits the advantages of the HT-FEM over the FEM and the BEM, such as high accuracy using coarse meshes, insensitivity to mesh distortion, great liberty in element shape, and accurately representing various local effects [4]. It is of interest to develop a uniform framework in HFS-FEM to solve anisotropic and isotropic problems together.

DEVELOPMENT OF THE METHODOLOGY

In the Cartesian coordinate system (x_1, x_2, x_3), if we neglect the body force b_i, the equilibrium equations, the stress-strain laws and the strain-displacement equations of for anisotropic elasticity are [2]

$$\sigma_{ij,j}=0, \qquad \sigma_{ij}=C_{ijkl}e_{kl}, \qquad e_{ij}=(u_{i,j}+u_{j,i})/2 \qquad i,j=1,2,3 \tag{1}$$

where σ_{ij} is the stress tensor, e_{kl} the strain tensor, C_{ijkl} the fourth-rank anisotropic elasticity tensor, and u_i the displacement vector. To solve the anisotropic problem governed by Eq. (1) using HFS-FEM approach, the solution domain Ω has to be divided into a series of elements as done in conventional FEM. For each element, two independent fields, i.e. intra-element field and frame field, are assumed in a manner as that presented in [5]. In this approach, the intra-element field $\mathbf{u(x)}$ and the frame field $\tilde{\mathbf{u}}(\mathbf{x})$ for a particular element $\mathbf{e}$ are approximated as

$$\mathbf{u(x)}=\mathbf{N_e c_e} \quad (\mathbf{x}\in\Omega_e, \mathbf{y}_{sj}\notin\Omega_e), \qquad \tilde{\mathbf{u}}(\mathbf{x})=\tilde{\mathbf{N}}_e\mathbf{d}_e \quad (\mathbf{x}\in\Gamma_e), \tag{2}$$

where the matrix $\mathbf{N_e}$ is composed by the fundamental solution $u^*_{ij}(\mathbf{x},\mathbf{y}_{sj})$, which is obtained by the Stroh formalism as $\mathbf{u}=\mathrm{Im}\left\{\mathbf{A}\left\langle\ln\left(z_\alpha-\hat{z}_\alpha\right)\right\rangle\mathbf{A^T}\right\}\hat{\mathbf{p}}/\pi$ [2], and unknown vector $\mathbf{c_e}=[c_{11} \quad c_{21} \quad c_{31} \quad \cdots \quad c_{1n} \quad c_{2n} \quad c_{3n}]^T$, $\tilde{\mathbf{N}}_e$ is the matrix of shape functions, $\mathbf{d}_e$ is the nodal displacements of elements. It is noted that the source point $\mathbf{y}_{sj}$ for the fundamental solution calculations are placed outside the element to avoid the singularity encountered as in BEM. In the above, complex variable $z_\alpha=x_1+p_\alpha x_2$, $\alpha=1,2,3$, p_α are material eigenvalues with positive imaginary part, $\mathbf{A}=[\mathbf{a_1},\mathbf{a_2},\mathbf{a_3}]$ and $\mathbf{B}=[\mathbf{b_1},\mathbf{b_2},\mathbf{b_3}]$ are 3×3 complex matrices composed by the material eigenvector matrix associated with p_α. In the absence of the body forces, the hybrid functional Π_{me} for a particular element $\mathbf{e}$ is constructed as

$$\Pi_{me}=\frac{1}{2}\iint_{\Omega_e}\sigma_{ij}\varepsilon_{ij}d\Omega-\int_{\Gamma_t}\bar{t}_i\tilde{u}_i d\Gamma+\int_{\Gamma_e}t_i(\tilde{u}_i-u_i)d\Gamma \tag{3}$$

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where the boundary Γ_e of the element is $\Gamma_e = \Gamma_{eu} \cup \Gamma_{et} \cup \Gamma_{eI}$, and where $\Gamma_{eu} = \Gamma_e \cap \Gamma_u, \Gamma_{et} = \Gamma_e \cap \Gamma_t$ and Γ_{eI} is the inter-element boundary of element **e**. Using Gaussian theorem and equilibrium equations, the stationary condition of the functional Π_{me} with respect to $\mathbf{c}_e$ and $\mathbf{d}_e$, respectively, yields the stiffness equation $\mathbf{K}_e\mathbf{d}_e = \mathbf{g}_e$, where the sparse element stiffness matrix $\mathbf{K}_e = \mathbf{G}_e^{\mathrm{T}}\mathbf{H}_e^{-1}\mathbf{G}_e$, and $\mathbf{H}_e = \int_{\Gamma_e} \mathbf{Q}_e^{\mathrm{T}}\mathbf{N}_e \mathrm{d}\Gamma$, $\mathbf{G}_e = \int_{\Gamma_e} \mathbf{Q}_e^{\mathrm{T}}\tilde{\mathbf{N}}_e \mathrm{d}\Gamma$, $\mathbf{g}_e = \int_{\Gamma_t} \tilde{\mathbf{N}}_e^{\mathrm{T}}\bar{\mathbf{t}} d\Gamma$ .The numerical calculations for $\mathbf{H}_e$, $\mathbf{G}_e$ and $\mathbf{g}_e$ can resort to the popular Gauss integration as used in FEM and BEM.

NUMERICAL EXAMPLES

An irregular composite lamina is modeled to demonstrate the performance of the proposed method in dealing with composite materials. The material parameters are assumed as E_1=11.8 *GPa*, E_2=5.9 *GPa*, G_{12}=0.69 *GPa*, v_{12}= 0.071. The geometry of the plate is shown in Fig.1 (a), where *L*=50 *mm* and *W*=50 *mm*, the semi-major and minor axes of the elliptical cut are *a*=2 *mm* and *b*=1 *mm*, respectively. A uniform tension of σ_0 =1 *GPa* is applied in x_2 direction.

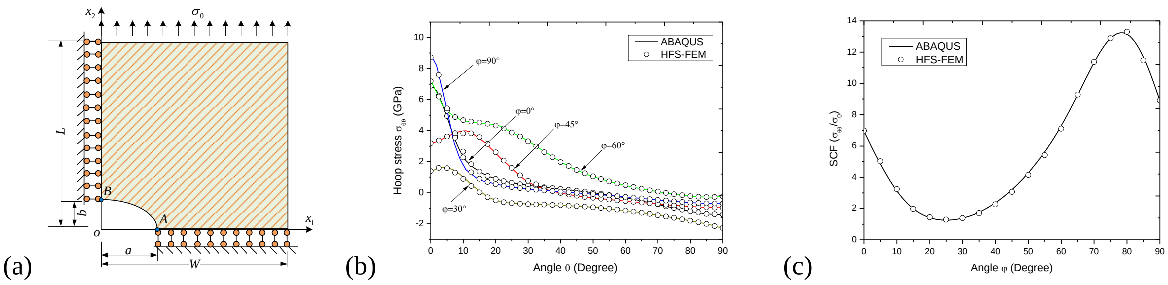

Fig.1: (a) Irregular composite plate with an elliptic cut; (b) Hoop stresses along the rim of the elliptical cut for different lamina angle φ; (c) Variation of stress concentration factor (SCF) at point *A* with fiber angle φ.

Fig. 1 (b) shows the hoop stresses along the rim of elliptical cut for different fiber angle φ. The relationship between SCF and fiber angle φ is described in Fig. 1 (c). It is obvious that the fiber orientation in lamina has a significant influence on the SCF of the plate. It is also observed that there is a very good agreement between the reference values by ABAQUS using a very fine mesh and the computed solutions by HFS-FEM on a much coarser mesh (about 1/5 of ABAQUS). These results indicate the capability of this method in analyzing static problems for anisotropic composite materials. Fig. 2 shows the contour plots of the stress components around the cut region when fiber angle $\varphi = 45^o$. Other important and interesting applications of the method will be presented in detail at the conference.

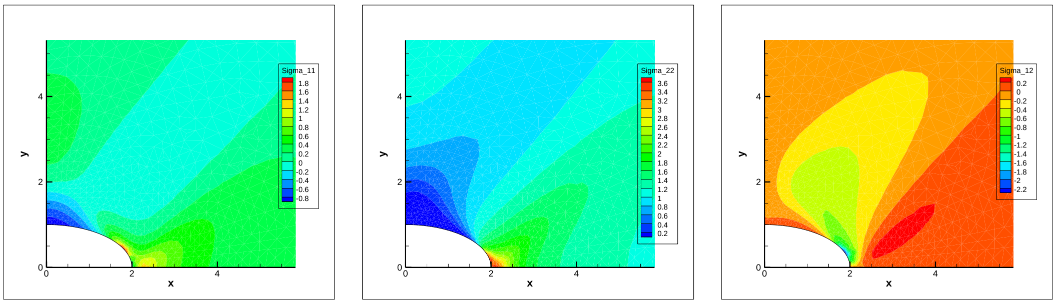

Fig. 2: Contour plots of stress components around the cut region in the composite plate (Lamina angle $\varphi = 45^0$).

CONCLUSIONS

A new hybrid finite element method, based on the fundamental solutions derived by Stroh formalism, has been developed to analyze anisotropic composite materials. Numerical results show that the proposed method is an accurate and efficient method to analyze various problems involving anisotropic composites, and it can be easily extended to analyze composite laminates. The detailed techniques involved and other applications of the method will be presented at the conference.

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