NECESSARY CONDITIONS AND APPROXIMATE SELF-CONSISTENT ESTIMATES FOR THE OVERALL CREEP FUNCTION OF VISCOELASTIC HETEROGENEOUS MATERIALS

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Summary In the context of linear viscoelastic composite materials, a unified approach is used to derive four exact asymptotic relations on the overall creep functions. It makes use of two expressions of the symbolic effective energy in the Laplace-Carson space. Based on these necesssary conditions, an approximation of self-consistent viscoelastic estimates is proposed. It is assessed by comparison with the customary collocation method and full-field simulations for various cases.

CONTEXT

This study adresses the problem of the effective creep response of viscoelastic heterogeneous materials made of Maxwellian constituents. Within a volume element of extension Ω, the local constitutive behaviour between the infinitesimal strain $\boldsymbol{\varepsilon}$ and the Cauchy stress $\boldsymbol{\sigma}$ thus reads, at each time t,

$$\dot{\boldsymbol{\varepsilon}}(\mathbf{x},t)=\mathbf{M}_e(\mathbf{x}):\dot{\boldsymbol{\sigma}}(\mathbf{x},t)+\mathbf{M}_v(\mathbf{x}):\boldsymbol{\sigma}(\mathbf{x},t),\ \forall\mathbf{x}\in\Omega, \tag{1}$$

with $\mathbf{M}_e(\mathbf{x})=\sum_r \chi_r(\mathbf{x})\mathbf{M}_e^{(r)}$ and $\mathbf{M}_v(\mathbf{x})=\sum_r \chi_r(\mathbf{x})\mathbf{M}_v^{(r)}$. $\mathbf{M}_e^{(r)}$ and $\mathbf{M}_v^{(r)}$ are respectively the elastic and viscous compliance tensors of phase (r) and $\chi_r(\mathbf{x})$ is its microstructural characteristic function (the volume fraction of phase (r) thus reads $c_r=\langle\chi_r\rangle$).

A distinctive feature of such composites is that the form of their overall response differs from the one of their local constitutive relation. In general, the macroscopic behaviour of the composite is not Maxwellian and exhibits a "long memory" effect [1]. Up to now, the derivation of approximate homogenized viscoelastic properties relied on the use of Prony series to represent the creep (resp. relaxation) function together with the collocation method [2, 3]. In the present study, another approximate model for the self-consistent (SC) estimate of the viscoelastic creep function is derived. It makes use of exact asymptotic relations similar to the ones previously obtained for the effective relaxation function [4].

NECESSARY CONDITIONS FOR THE OVERALL CREEP FUNCTION

By assuming a relaxed and undeformed state at $t=0$, the Laplace-Carson (LC) transform of relation (1) can be expressed in the two equivalent following forms, $\forall\mathbf{x}\in\Omega$,

$$\begin{cases} \boldsymbol{\varepsilon}^*(\mathbf{x},p)=\mathbf{M}^*(\mathbf{x},p):\boldsymbol{\sigma}^*(\mathbf{x},p) & \text{with}\quad \mathbf{M}^*(\mathbf{x},p)=\mathbf{M}_e(\mathbf{x})+\dfrac{1}{p}\mathbf{M}_v(\mathbf{x}), \\ \dot{\boldsymbol{\varepsilon}}^*(\mathbf{x},p)=\mathcal{M}^*(\mathbf{x},p):\boldsymbol{\sigma}^*(\mathbf{x},p) & \text{with}\quad \mathcal{M}^*(\mathbf{x},p)=p\,\mathbf{M}^*(\mathbf{x},p). \end{cases} \tag{2}$$

Based on these constitutive relations, the overall energy of the viscoelastic composite can be evaluated in the LC space for a constant macroscopic stress $\overline{\boldsymbol{\sigma}}$. By considering the expression of its partial derivative with respect to the transform variable p for the two asymptotic cases ($p=0$ and $p\to+\infty$), four necessary conditions are obtained for the overall creep functions

$$\begin{cases} \widetilde{\mathbf{M}}^*(+\infty)=\widetilde{\mathbf{M}}_e, \qquad \widetilde{\mathcal{M}}^*(0)=\widetilde{\mathbf{M}}_v, \\ \overline{\boldsymbol{\sigma}}:\dfrac{\partial\widetilde{\mathbf{M}}^*}{\partial(1/p)}(+\infty):\overline{\boldsymbol{\sigma}}=\displaystyle\sum_r c_r\,\mathbf{M}_v^{(r)}::\langle\boldsymbol{\sigma}_e\otimes\boldsymbol{\sigma}_e\rangle_r\,, \\ \overline{\boldsymbol{\sigma}}:\dfrac{\partial\widetilde{\mathcal{M}}^*}{\partial p}(0):\overline{\boldsymbol{\sigma}}=\displaystyle\sum_r c_r\,\mathbf{M}_e^{(r)}::\langle\boldsymbol{\sigma}_v\otimes\boldsymbol{\sigma}_v\rangle_r \end{cases} \tag{3}$$

with $\boldsymbol{\sigma}_e(\mathbf{x})$ and $\boldsymbol{\sigma}_v(\mathbf{x})$ denoting the stress fields solutions of the purely elastic and viscous problems respectively. The two first relations are the well-known asymptotic elastic and viscous responses at the overall scale [3] while the two last relations are new. They can be considered as the dual form of the relations given in [4] for the effective relaxation function.

APPROXIMATE MODEL FOR ISOTROPIC INCOMPRESSIBLE MATERIALS

By making use of the relations (3), an approximate model with solely two retardation times can be derived to approximate the response of isotropic incompressible composites whose effective compliance tensor reads

$$\widetilde{\mathbf{M}}(t)=\widetilde{m}(t)\mathbf{K} \tag{4}$$

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with $\mathbf{K}$ the usual deviatoric projector. According to the proposed model, the effective scalar creep function may be written as

$$2\widetilde{m}(t) = \widetilde{m}_e + t\,\widetilde{m}_v + \overline{m}'\left[1 - \exp\left(-t/\overline{\tau}'\right)\right] \tag{5}$$

with $\widetilde{m}_e = 1/\widetilde{\mu}_e$ and $\widetilde{m}_v = 1/\widetilde{\mu}_v$ the effective elastic and viscous shear compliances and the parameters $\overline{\tau}'$ and $\overline{m}'$ are defined as:

$$\frac{\overline{m}'}{\overline{\tau}'} = \sum_r m_v^{(r)} \frac{\partial\,\widetilde{m}_e}{\partial\,m_e^{(r)}} \;-\; \widetilde{m}_v \quad \text{and} \quad \overline{m}' = \sum_r m_e^{(r)} \frac{\partial\,\widetilde{m}_v}{\partial\,m_v^{(r)}} \;-\; \widetilde{m}_e \tag{6}$$

with $m_e^{(r)} = 1/\mu_e^{(r)}$ and $m_v^{(r)} = 1/\mu_v^{(r)}$ the elastic and viscous shear compliances of phase (r). One of the two retardation times is simply the one corresponding to the Maxwellian approximation of the macroscopic behaviour.

It has been previously demonstrated that the proposed model is exact for the particular case of the Hashin-Shtrikman estimate for two-phase composite and delivers a good approximation for the checkerboard problem [4]. For this latter case, the composite exhibits at the macroscopic scale a continuous relaxation (resp. retardation) spectrum whose analytical expression has been recently given [5]. Following these results, the approximate model is here assessed in the context of self-consistent estimates which are also known to exhibit a continuous spectrum [6].We consider various situations for which an analytical expression of the elastic SC estimate is available. An illustrative example is given in figure 1 for polycrystals with hexagonal-close-packed crystalline structure. Under creep loadings at two stress levels, the model compares well with full-field FFT computations as well as with the results obtained with the collocation method. The inadequacy of the Maxwell model is shown.

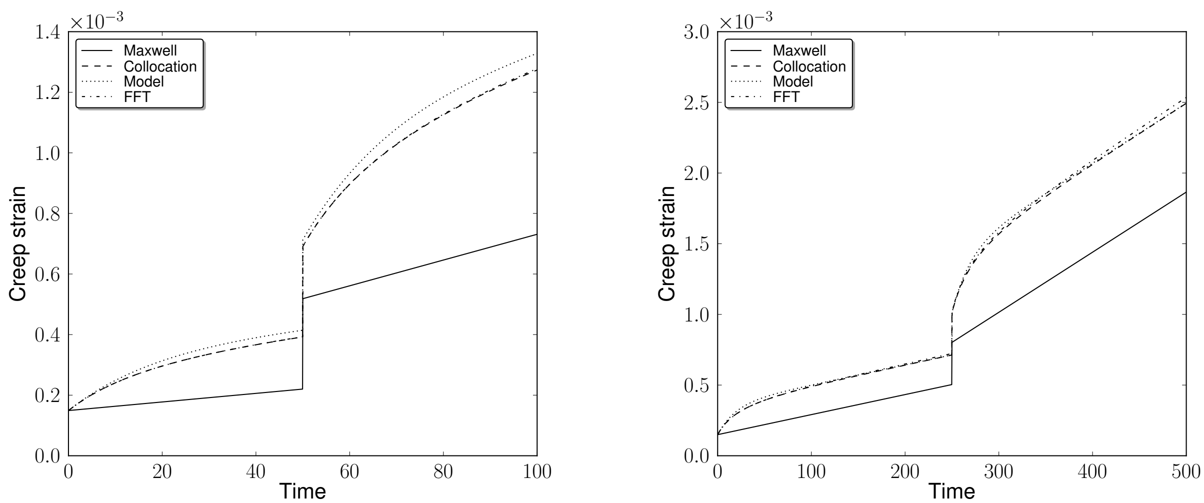

Figure 1. Overall creep response of an isotropic incompressible polycrystal made of grains with hexagonal-close-packed crystalline structure. Comparison between full-field simulations (FFT), the Maxwell model with a single retardation time, the approximate model with two retardation times and the collocation method with five retardation times. Left: creep test focusing on the transient regime. Right: creep test with transient and steady-state regimes.

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