Dynamic Responses of an Elastic Ring on Viscoelastic Foundation to External Impact

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Summary This paper studies the dynamic responses of an elastic Ring on ViscoElastic Foundations (RoVEF). The model comes from a flexible crashworthy structure that protects bridge pile against ship impact. The time-dependent response of the RoVEF to external impact is controlled by a governing Partial Differential Equation (PDE) with adequate initial and boundary conditions. The initial-boundary value problem (IBVP) is solved by using the Laplace Transform (LT) and the numerical inverse transform technique. The solutions in the LT domain are expressed in exact forms, and the final dynamic responses of the RoVEF in the time domain are evaluated by using the numerical inverse transform software. The current paper reports the fundamental solution procedures, as well as the major results of the dynamic responses of the RoVEF.

INTRODUCTION

A flexible energy-dissipating crashworthy structure has been developed to protect the bridge against ship impact [1]. The structure (Fig 1a) is composed of a steel inner ring, a steel outer ring, and many steel wire rope coils connecting the two rings as cushions. The structure can be modelled as a Ring-on-Foundation (RoF). Previous work studied the quasistatic behavior of the RoF [2]. The current work studies the dynamic responses of the structure under an external impact force. The viscous behavior of the cushion elements is also included in the study.

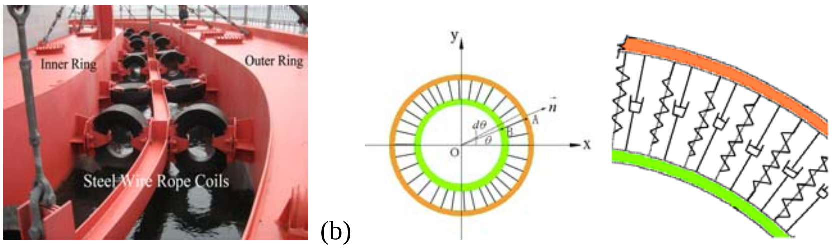
(a)  (b)

Fig 1: (a) A flexible ship-pile crashworthy structure; (b) RoVEF model

MODEL AND GOVERNING EQUATION

The crashworthy structure is modeled as an elastic ring of radius R which is linked to the foundation by viscoelastic connectors (Fig 1b). These connectors are idealized as viscoelastic foundations with elastic and viscous coefficients per unit angle k and η. Treated as a curved beam with the bending stiffness EI and line mass density ρ, the RoVEF deforms with its radial displacement u expressed as a function of angle θ. This $u(\theta)$ function satisfies:

$$u^{(5)}(\theta,t)+2u^{(3)}(\theta,t)+\left(1+\frac{kR^3}{EI}\right)u'(\theta,t)+\frac{\rho R^4}{EI}\left[\frac{\partial}{\partial\theta}\left(\frac{\partial^2 u(\theta,t)}{\partial t^2}\right)\right]+\frac{\eta R^4}{EI}\left[\frac{\partial}{\partial\theta}\left(\frac{\partial u(\theta,t)}{\partial t}\right)\right]=0$$

Using the characteristic time $T=\sqrt{\rho R^4/EI}$ and length R to nondimensionize the variables: $\bar{u}=u/R$, $\bar{t}=t/T$, the governing equation is reduced to:

$$\frac{\partial^5\bar{u}(\theta,\bar{t})}{\partial\theta^5}+2\frac{\partial^3\bar{u}(\theta,\bar{t})}{\partial\theta^3}+\left(1+\lambda^2\right)\frac{\partial\bar{u}(\theta,\bar{t})}{\partial\theta}+\frac{\partial^3\bar{u}(\theta,\bar{t})}{\partial\theta\partial\bar{t}^2}+\bar{\eta}\frac{\partial^2\bar{u}(\theta,\bar{t})}{\partial\theta\partial\bar{t}}=0 \quad (1)$$

where $\lambda=\sqrt{kR^3/EI}$ is the stiffness ratio of the foundation to the ring, and $\bar{\eta}=\eta T/\rho$ the nondimensional viscosity. Under external impact loading F_0, (1) must be solved with following Initial and Boundary Equations:

$$\begin{aligned}&\bar{u}(\theta,0)=\frac{\partial\bar{u}}{\partial t}(\theta,0)=0,\ \frac{\partial\bar{u}}{\partial\theta}(0,\bar{t})=\frac{\partial\bar{u}}{\partial\theta}(\pi,\bar{t})=0,\ \int_0^\pi\bar{u}(\theta,\bar{t})\,d\theta=0\\&\frac{\partial\bar{u}}{\partial\theta}(\pi,\bar{t})+\frac{\partial^3\bar{u}}{\partial\theta^3}(\pi,\bar{t})=0,\ \frac{\partial\bar{u}}{\partial\theta}(0,\bar{t})+\frac{\partial^3\bar{u}}{\partial\theta^3}(0,\bar{t})=\pi\lambda^2\bar{F}_0\mathbf{H}(\bar{t})\end{aligned} \quad (2)$$

where $\bar{F}_0=F_0/2\pi kR$ is the non-dimensionalized applied force, and $\mathbf{H}(t)$ the Heaviside function.

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SOLUTION STRATEGY AND MAJOR RESULTS

We use Laplace transform to solve (1, 2). Define: $\tilde{u}(\theta,s)=\mathbf{L}\left[\bar{u}(\theta,\bar{t}),\bar{t}\to s\right]=\int_0^\infty \bar{u}(\theta,\bar{t})e^{-s\bar{t}}d\bar{t}$ (3)

The PDE is changed into an ODE, with s being a parameter:

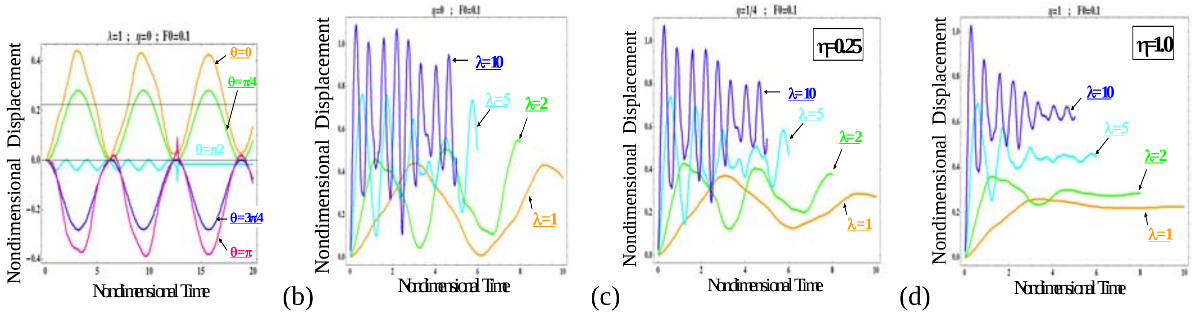
$$\tilde{u}^{(5)}(\theta,s)+2\tilde{u}^{(3)}(\theta,s)+\left(1+\lambda^2+\bar{\eta}s+s^2\right)\tilde{u}'(\theta,s)=0$$
$$\tilde{u}'(0,s)=0\ ,\ \tilde{u}'(\pi,s)=0\ ,\ \tilde{u}'(\pi,s)+\tilde{u}^{(3)}(\pi,s)=0\ ,\ \tilde{u}'(0,s)+\tilde{u}^{(3)}(0,s)=\pi\lambda^2\bar{F}_0/s\ ,\ \int_0^\pi \tilde{u}(\theta,s)d\theta=0 \tag{4}$$

which has solution:

$$\tilde{u}(\theta,s)=\frac{\pi\lambda^2F_0}{2\sqrt{\lambda^2+\bar{\eta}s+s^2}\sqrt{1+\lambda^2+\bar{\eta}s+s^2}\,s}\left[-\frac{2\sqrt{\lambda^2+\bar{\eta}s+s^2}}{\sqrt{1+\lambda^2+\bar{\eta}s+s^2}\,\pi}+\frac{(\alpha i+\beta)\left(e^{(\alpha+i\beta)(\theta-\pi)}+e^{(\alpha+i\beta)(\pi-\theta)}\right)}{e^{(\alpha+i\beta)\pi}-e^{-(\alpha+i\beta)\pi}}-\frac{(\alpha i-\beta)\left(e^{(\alpha-i\beta)(\theta-\pi)}+e^{(\alpha-i\beta)(\pi-\theta)}\right)}{e^{(\alpha-i\beta)\pi}-e^{-(\alpha-i\beta)\pi}}\right] \tag{5}$$

where $\alpha(s)=\sqrt{\left(\sqrt{1+\lambda^2+\bar{\eta}s+s^2}-1\right)/2}$, $\beta(s)=\sqrt{\left(\sqrt{1+\lambda^2+\bar{\eta}s+s^2}+1\right)/2}$. The exact solution $\tilde{u}(\theta,s)$ in the transformed domain is an explicit function of the s. Numerical inverse technique is used to obtain the original function $\bar{u}(\theta,\bar{t})$ in the time domain. The fixed Talbot algorithm integrated as a concise MATHEMATICA subroutine in [3] is used for the numerical calculation.

Fig 2a plots the time histories of the radial deflections at different sites. These curves are periodic with an approximate period 2π, meaning that the period of the structure is equivalent to the time the bending wave propagates one round through the ring. The deformation of the rings with λ =1～10 are shown in Fig 2b, where $\bar{\eta}=0$ (no damping). As λ increases, the ring becomes more flexible and the deformation is more localized. Fig 2c, 2d depicts the displacement histories of the RoVEF with $\bar{\eta}=0.25$ and 1.0. The damping effect is more significant as $\bar{\eta}$ increases.



(a) (b) (c) (d)

Fig 2 (a) Vibrations of a undamped RoVEF; (b)~(c) Ddeflections of RoVEF with different λ and $\bar{\eta}$ values.

CONCLUSIONS

The current work studies dynamic responses of a RoVEF. The results show that the dynamic responses of the RoVEF are significantly affected by the parameter λ. For $\lambda<1$ case, the vibration of the ring is basically reciprocating translation around the quasistatic equilibrated position. The ring acts like a rigid structure and the bending deformation is negligible. For the large λ case, the deformation of the ring becomes complex. The ring's motion includes both the bending wave propagation and the global translation. The existence of $\bar{\eta}$ attenuates the dynamic oscillations, leading the long term deformation of the RoVEF to the quasistatic equilibrated position.

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References

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