PERTURBATION METHOD IN ONE-DIMENSIONAL DYNAMICS OF AN INCOMPRESSIBLE ELASTIC MEDIUM UNDER IMPACT LOADING

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Summary The main qualitative features of the processes of one-dimensional shear shock deformation are described by solving the corresponding evolution equation. This equation defines behaviour of the solution in the near-front area of wave process and follows from interior lines of a method of matched asymptotic expansions. The report presents the results for many boundary value problems with a flat and axial symmetry for the case of one shock wave.

The motion of shock waves in solids studied much less than in the mechanics of liquids and gases. First of all, this applies to the transverse and quasitransverse wave processes. For a compressible nonlinearly elastic medium the motion of shear waves is always preceded by volume deformation, which is reflected in the character of transverse wave, changing it to the quasitransverse wave[1]. This complicates the mathematical description of these waves. Another difficulty is related to the dependence of the velocity of shock waves on the state of the medium in their vicinity [1]. As a result, in general, is unknown in advance of the shock wave geometry and its position. They can be determined only jointly with the other unknowns of the problem. These problems lead to the need to resort to approximate numerical or analytical methods of the solution. At present one of the most effective analytical methods should be considered the method of matched asymptotic expansions [2], or variant of the ray method for shock waves [3]. In this paper the authors apply the method of matched asymptotic expansions to one-dimensional boundary value problems in incompressible nonlinear elastic medium, in which the boundary conditions lead to the appearance of shock waves. The model of an incompressible medium is chosen solely for the ability to describe the purely transverse effects of dynamic deformation.

In the representation of the Euler Cartesian space coordinates $x_i\ (i=1,2,3)$, closed system of equations determining the motion of the medium are written as

$$v_i=\dot{u}_i+u_{i,j}v_j,\quad \alpha_{ij}=\frac{1}{2}\left(u_{i,j}+u_{j,i}-u_{k,i}u_{k,j}\right),\quad \sigma_{ij,j}=\rho\left(\dot{v}_i+v_{i,j}v_j\right),\quad \sigma_{ij}=-p\delta_{ij}+\frac{\partial W}{\partial \alpha_{ik}}\left(\delta_{kj}-2\alpha_{kj}\right),$$
$$W\left(I_1,I_2\right)=\left(a-\mu\right)I_1+aI_2+bI_1^2-\kappa I_1I_2-\theta I_1^3+cI_1^4+dI_2^2+kI_1^2I_2+\ldots,\quad I_1=\alpha_{ii},\quad I_2=\alpha_{ij}\alpha_{ji}. \tag{1}$$

where u_i and v_i are the components of the displacement and velocity vectors, α_{ij} are the components of the Almansi strain tensor, σ_{ij} are the components of the Euler-Cauchy stress tensor, $\rho = const$ is the density of the medium, p is the an additional hydrostatic pressure, W is an elastic potential function, $a, \mu, b, \kappa, \theta, c, d, k$ are the elastic constans, δ_{ij} is the Kronecker delta.

For an elastic half-space bounded by the plane $x_1=0$, the problem with the boundary condition

$$u_2\big|_{x_1=0,\,t\geq 0}=g\left(t\right), \tag{2}$$

is considered. Here $g(t)$ is a known function, which leads immediately or subsequently to the formation of a shock wave $(\Sigma(t))$. On the wave $\Sigma(t)$ conditions are given by

$$X_1\left(t\right)=\int_{t_0}^{t}G\left(\xi\right)d\xi,\quad G=C\left(1+\alpha_1\gamma^2+\ldots\right),\quad \alpha_1=\frac{a+b+\kappa+d}{\mu},$$
$$\gamma=\left[u_{2,1}\right],\quad C=\sqrt{\mu\rho^{-1}},\quad u_2\big|_{x_1=X_1(t)}=0,\quad \left[u_{2,1}\right]\Big|_{X_1(t)}=-u_{2,1}^{-}\Big|_{X_1(t)}. \tag{3}$$

In the distant from the border area we introduce the dimensionless variables

$$n=\varepsilon^2x_1C^{-1}T^{-1},\quad p=\left(x_1-Ct\right)C^{-1}T^{-1},\quad w\left(n,\,p\right)=\varepsilon^{-1}u_2\left(x_1,t\right)C^{-1}T^{-1},\quad w=w_0+\varepsilon^2w_2+\ldots, \tag{4}$$

where ε is a small parameter of the problem, T is a characteristic time. In the zero step of the method we obtain the evolution equation of the shear waves:

$$v_{0,n}+1{,}5\alpha v_0^2v_{0,p}=0,\quad w_{0,p}=v_0. \tag{5}$$

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The authors consider of the various methods of representing the solution of this equation, allowing to build the displacement field for different conditions (2) on the basis of its solution. A method of solution for subsequent approximations $(w_2, w_4, \ldots)$ is submitted in the report.

In addition to the plane case, the report provides a transition to problems with axial symmetry in the two examples of problems of antiplane motion of boundary of a cylindrical cavity in an incompressible medium and twisting motion of the boundary of this cavity. In both cases, the perturbation method in the near-front area of shock waves leads to an equation of (5) type, but with additional terms that determine the attenuation of the solution.

The most interesting problem with axial symmetry for the previously undeformed medium occurs when the boundary points of the cavity $(r = r_0)$ are moving according to the law

$$r = r_0, \quad \varphi = \varphi_0(t), \quad u_z = z_0(t), \tag{6}$$

where r, φ, z are the cylindrical coordinates, $\varphi_0(t)$ and $z_0(t)$ are known functions. In this case, assuming that all points of the medium make the helical motion, we define the kinematics as

$$u_r(r,t) = r(1-\cos\varphi), \quad u_\varphi(r,t) = r\sin\varphi, \quad u_z(r,t), \quad \varphi(r,t). \tag{7}$$

Analysis of the dynamic compatibility conditions in this case leads to two possible types of shock waves. Their speeds are:

$$G_1 = C\left\{1+\alpha\left(r^2\left[\varphi_{,r}\right]^2+\left[u_{z,r}\right]^2\right)+\ldots\right\}^{1/2}, \quad G_2 = C\left\{1+\alpha\left(r^2\left(\varphi_{,r}^+\right)^2+\left(u_{z,r}^+\right)^2\right)+\ldots\right\}^{1/2}. \tag{8}$$

The first wave changes only the intensity of the initial shear without change of direction. On the second wave is $\left[r^2\varphi_{,r}^2+u_{z,r}^2\right]=0$ and only the direction of shear change. Application of the method of small parameter with the definition of the dimensionless functions h and w, corresponding to functions φ and u_z, and the choice of half-characteristic dimensionless variables n, p in region, that remote from $r = r_0$, leads to a system of evolution equations:

$$\begin{gathered} 2a_{0,n}+\alpha\left(3(1+n)^2 a_0^2+\right)a_{0,p}+2\alpha a_0 b_0 b_{0,p}+\frac{3a_0}{1+n}=0, \\ 2b_{0,n}+\alpha\left((1+n)^2 a_0^2+3b_0^2\right)b_{0,p}+2\alpha(1+n)^2 a_0 b_0 a_{0,p}+\frac{b_0}{1+n}=0, \quad a_0 = h_{0,p}, \; b_0 = w_{0,p}. \end{gathered} \tag{9}$$

Along each of the families defined Riemann invariants. Their structure is consistent with the previous conclusions about the possible types of shock waves.

The presented solutions demonstrate the effectiveness of the perturbation method applied to time-dependent dynamics of elastic media. The results can be generalized and extended to the case of multidimensional problems, provided that the half- characteristic dimensionless coordinates are based on the ray grid.

References

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