BAND GAPS OF PHONONIC CRYSTALS WITH SMOOTH CONTACT INTERFACES

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Summary The boundary element method (BEM) is presented to compute the band gaps of two-dimensional (2D) phononic crystals with perfectly contact or smoothly contact interfaces between solid the scatterers and the solid matrix. The boundary integral equations are derived in a unit cell. By considering the interface conditions and the Bloch conditions, a linear eigenvalue problem which involves small matrices is formulated. Numerical results shows that the present method is effective and accuracy, and the band gaps can be adjusted by the interface conditions. Compared with the perfectly contact interface, the band gap of the smooth contact interface is lower and narrower.

INTRODUCTION

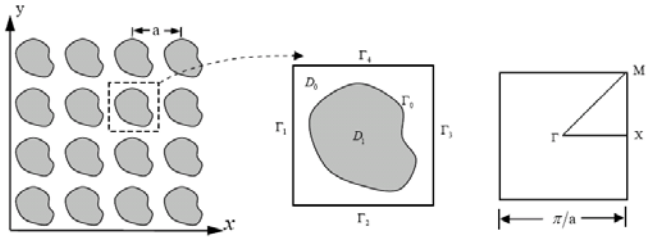
Elastic band gap materials called "phononic crystals" are inhomogeneous elastic media composed of one-, two-, or three-dimensional periodic arrays of scatterers embedded in a matrix. Several classes of phononic crystals exist and differ mainly by the physical nature of the scatterer and the matrix. These composite media typically exhibit stop bands for which the propagation of sound or vibration is strictly forbidden in all directions. The existence of absolute band gaps has been investigated both theoretically and experimentally [1-3]. Such materials should have many potential applications as elastic-acoustic filters, mirror or transducers. Since the phononic band gap (PBG) is the major feature of a PBG structure, it is essential to design a structure which possesses a band gap as large as possible. The most severe limit to the PBG size comes from the degeneracy of phononic bands at high symmetry points in the Brillouin zone. Consequently, many approaches have been proposed to lift the band degeneracy and thus enlarge the PBG. However, some methods cannot consider the interface conditions so that the interface influences on band gaps are neglected. Here we develop a boundary element method to compute the band gaps of two-dimensional solid phononic crystals with different perfectly or smoothly contact interface conditions.

THEORY AND FORMULATION

We consider a 2D phononic crystal consisting of a square array with the lattice constant a, see Fig.1. Both scatterers and matrix are homogeneous, isotropic and linear elastic solids. The governing equations describing these time-harmonic wave motions can be expressed as

$$\lambda \frac{\partial}{\partial x_\alpha}\left(\frac{\partial u_\beta}{\partial x_\beta}\right)+\mu \frac{\partial}{\partial x_\beta}\left[\frac{\partial u_\alpha}{\partial x_\beta}+\frac{\partial u_\beta}{\partial x_\alpha}\right]+\rho\omega^2 u_\alpha = 0 \qquad (1)$$

where $\alpha, \beta = x, y$; the usual summation convention is assumed for the repeated indices; $\nabla = (\partial/\partial x, \partial/\partial y)$; ω is the angular frequency; λ, μ and ρ are the Lamé constants and mass density of the solids, respectively.



(a.1) (a.2) (a.3)

Fig.1 The structures under consideration: (a.1), (a.2) and (a.3) are the square lattice, its corresponding square unit cell and the first Brillouin zone, respectively

Due to the periodicity of the system, we can restrict our attention to a unit cell (Fig.1(a2)). For the solid/solid system, when the binary components are perfectly contact, the boundary conditions along the interface are as follows

$$\mathbf{u}_1(\mathbf{r}) = \mathbf{u}_0(\mathbf{r}), \quad \mathbf{T}_1(\mathbf{r}) = \mathbf{T}_0(\mathbf{r}), \quad \mathbf{r} \in \Gamma_0 \qquad (2)$$

When the solid inclusion and solid matrix have a smooth contact, the boundary conditions along the interface are

$$u_1^{\,n}(\mathbf{r}) = u_0^{\,n}(\mathbf{r}),\ T_1^{\,n}(\mathbf{r}) = T_0^{\,n}(\mathbf{r}), T_1^{\,t}(\mathbf{r}) = T_0^{\,t}(\mathbf{r}) = 0, \quad \mathbf{r} \in \Gamma_0 \qquad (3)$$

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where u_0^n, T_0^n, T_0^t and u_1^n, T_1^n, T_1^t are the normal displacement, normal and tangitudinal traction components of the matrix and the scatterer.

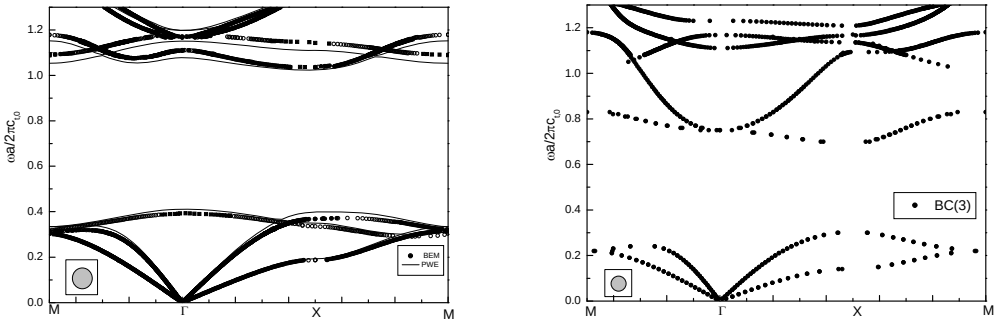
In addition, the periodicity of the structures implies that any field quantity ϕ (e.g. the displacement, stress, pressure and their derivatives) should obey the following Bloch theorem

$$\phi(x,y) = e^{i(k_x x + k_y y)} \Phi(x,y) \qquad (4)$$

where (k_x, k_y) is a real Bloch wave vector and $\Phi(x,y)$ follows the same periodic condition as the structures. The Bloch theorem allows us to solve the problem in a unit cell. It should be noted that the most developed methods, such as PWE method, wavelet method and FDTD method et al. cannot deal with the different interface conditions. The present method accounts for these conditions and formulate the linear eigenvalue equations. The details can be referred to Ref. [2].

NUMERICAL RESULTS AND DISCUSSIONS

We consider a square lattice of aurum (Au) cylinders embedded in the epoxy matrix. The density and the wave velocities of the component materials are: $\rho_1 = 19500\text{kg/m}^3$, $c_{l,1} = 3360\text{m/s}$ and $c_{t,1} = 1239\text{m/s}$ for the Au; and $\rho_2 = 1180\text{kg/m}^3$, $c_{l,2} = 2540\text{m/s}$ and $c_{t,2} = 1160\text{m/s}$ for the epoxy. The lattice constant is $a = 3.0\text{mm}$ and the filling ratio is $f = 0.4$. We respectively discretize the exterior and inner boundaries as 32 and 48 constant elements. The band structures of the in-plane mixed wave modes with the interface condition (2) are demonstrated in Fig. 2(a) by the scattered dots, from which there is one complete band gap [0.388, 1.028]. For comparison, the band structures computed by the PWE method with 961 plane waves are also plotted in the figure by the solid lines. It can be observed that the results obtained by the two methods show good agreement in the low frequency region but are slightly different in the high frequency region. However, the results obtained by the present BEM are nearly identical to those computed by the wavelet method [1]. This example verifies that the BEM is effective and accurate for band gap computations of phononic crystals, and it involves small matrices and less time-consuming. For the smooth contact interface (3), Fig. 2(b) shows the in-plane band structures by the scattered dots. The bands are all lower than those of the perfectly contact interface, the forms of which are completely different. Form Fig. 2(b), there is also one complete band gap [0.305, 0.710], both the lower and upper edges of which are lower than those of the perfectly contact interface condition (2) and the obtained band gap width is narrower. It is shown that the band gaps can be controlled not only by the material parameters and the structure geometrical parameters but also by the interface conditions. This provides us an alternative way for the design of the phononic crystals with certain band gaps.



(a) (b)

Fig.2 In-plane band structures of the aurum/epoxy phononic crystal in a square lattice: (a) The scattered dots and solid lines represent the results of the interface condition (2) obtained by BEM and PWE method (961 plane waves), respectively; (b).The scattered dots represent the results of interface conditions (3) computed by BEM.

References

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