

# SIMILARITY IN STRUCTURES SUBJECTED TO IMPACT LOADS WHEN MODEL IS DISTORTED IN GEOMETRY AND MATERIAL

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**Summary** In the present manuscript, strain rate sensitive scaled structures with distorted geometry and material are used to predict the behaviour of the corresponding prototype. These three distortions – strain rate, distorted geometry and material - infringe the  $\Pi$  theorem requirements for complete similarity. In order to overcome this limitation, a scaling method which considers these effects is applied so yielding a model whose final deformation is similar to the prototype.

## INTRODUCTION

Many works have reported incomplete similarity in impact tests carried on scaled structures (models or replicas) [1], [2]. The response of those models diverges from the behaviour of the corresponding full-size compound (prototype). In the present study, three effects that generate incomplete similitude are considered: strain rate in scaled structures, non-proportional scaling in one of the dimensions of the model, and quasi-static yielding stress of replica different from the prototype. These three features infringe the  $\Pi$  theorem requirements for complete similarity. Accordingly, the standard scaling laws do not provide the means to solve this problem. In order to overcome this limitation, a method which takes these effects into account is applied so generating a replica whose response can be compared with the prototype. An analytical and a numerical model are used to analyse the method.

## METHODOLOGY

Differently from the standard scaling laws, which employ the same impact velocity for model and prototype [3], the method applied in the present work changes the impact velocity in a rational way so leading to a replica geometrically similar to the full-size structure. The velocity factor,  $\beta_v = V_{\text{model}}/V_{\text{prototype}}$ , is given by

$$\beta_v = \underbrace{\beta^{q/(q-2)}}_{f_1} \underbrace{(\beta_x/\beta)^{n_v}}_{f_2} \underbrace{\beta_\sigma^{1/(2-q)}}_{f_3}, \quad (1)$$

being  $\beta$  the geometry scaling factor,  $q$  the exponent parameter for the Norton constitutive equation [4],  $\beta_x$  the scaling factor for the dimension of the structure that is not proportionally scaled,  $n_v$  an exponent that is calculated by the method, and  $\beta_\sigma$  the ratio between the quasi-static yielding stress of model and prototype. The first term,  $f_1$ , is the factor that expresses the influence of strain rate in the model response [5].  $f_2$  defines the effect of the non-proportional scaling factor [6].  $f_3$  expresses the distortion of the material [7].

An important characteristic of the method is that the full-size structure behaviour is predicted by using the model response. The exponent  $n_v$  is calculated without any data regarding the prototype.

The scaling factors for the other variables of the phenomenon (acceleration, stress, time, energy, etc.) can also be calculated, and the entire model behaviour can be related to the prototype.

## RESULTS

### Analytical model

The analytical problem represents a rectangular cross-section beam clamped at both ends and struck at mid-span by a rigid mass with an initial velocity  $V_0$ . The results for the dimensionless final displacement at the impact point,  $w_L$ , are summarized in Table 1.

Table 1. Results for the problem of a clamped beam struck by a mass at mid-span.

|                   | distorted dimension | $\beta$ | $\beta_x/\beta$       | $\beta_v$ | $w_L$  | difference to the prototype ( $w_L$ ) |
|-------------------|---------------------|---------|-----------------------|-----------|--------|---------------------------------------|
| prototype         | -                   | 1       | -                     | 1         | 0.7634 | -                                     |
| uncorrected model | length              | 0.1     | $\beta_x/\beta = 2.0$ | 1         | 1.0340 | 35.44%                                |
| corrected model   | length              | 0.1     | $\beta_x/\beta = 2.0$ | 0.7235    | 0.7446 | 2.47%                                 |

### Numerical model

It is analysed the problem of a double hull ship panel subject to the impact of a rigid mass. The structure is constituted by 18 mm thick steel plates, and it is struck horizontally by a rigid mass with spherical end that moves perpendicularly to the main plane of the panel. The adopted geometry is shown in Figure 1 (a) and (b), and it is based in a simplified representation of a double hull DWT 150K. The internal nodes of the bottom and the deck are clamped. The material of the structure is ruled by the Norton constitutive equation with  $\sigma_0 = 450$  MPa,  $\dot{\epsilon}_0 = 1.0\text{e-}3 \text{ s}^{-1}$ , and  $q = 0.077$ , and an elastic modulus of 210 GPa and hardening of 50 MPa. The model is scaled by  $\beta = 0.25/18$ , but a geometric distortion in thickness of the model is considered. Instead of 0.25 mm (as required by the scaling laws), the model is constituted by plates of 0.50 mm thickness, i.e.  $\beta_x/\beta = 2.0$ . Figure 1 (c) compares the horizontal displacement at the impact point,  $\delta$ , of the models with the prototype.

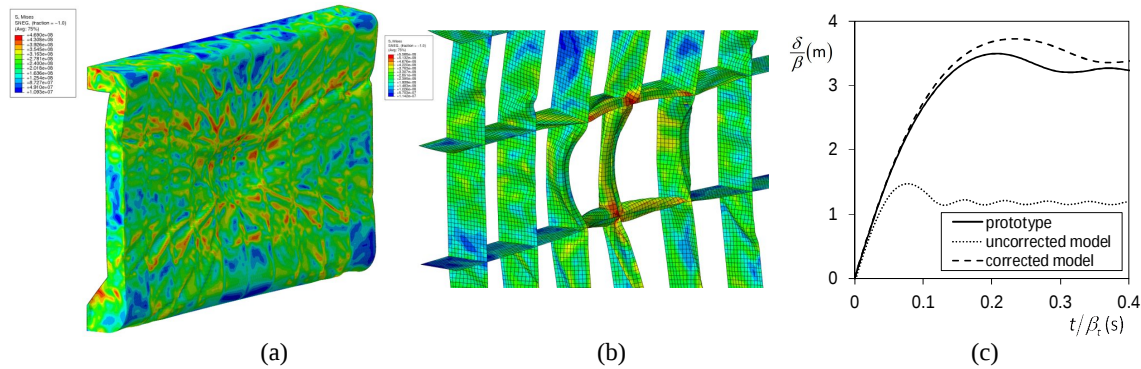


Figure 1. Results for the double hull panel struck by a mass. (a) final deformed configuration. (b) internal frame. (c) horizontal displacement at the impact point.

## CONCLUSIONS

It was analysed a method which can handle with restrictions that the usual scaling laws cannot overcome through the use of two examples. In the first problem, the analytical model showed a significant improvement in the results when the velocity was changed according to Eq. (1). The error in the final displacement at mid-point of the replica beam decreases from 35.44% (uncorrected model) to 2.47% (corrected model) when compared to the prototype. The displacement at the impact point in the second problem is shown in Figure 1 (c). The corrected model is clearly more similar to the prototype response than the uncorrected replica. The final displacement of the uncorrected model is about 1/3 of the prototype one, whereas the corrected model exhibits a difference of 3.20%.

## References

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