THE STAR SHAPED CRACK PATTERNS OF BROKEN WINDOWS

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Summary Broken windows typically present a pattern of radial cracks that extend outwards from the impact point. Such patterns of multiple cracks remain difficult to interpret, but they are of primary importance for the understanding of impact fragmentation. Performing controlled impacts on PMMA plates, we study the variation of the pattern with the impact parameters. The number of radial cracks varies with the impact speed and we propose a quasistatic model that is consistent with our experimental results.

PHENOMENOLOGY

Numerous studies on the behavior of a plate impacted by a projectile have been published in relation to security applications [1]. In this context, the emphasis is usually put on the damage and perforation of armour plate made of high strength ductile materials. Different perforation mechanisms leading to different *post-mortem* shapes have been identified and studied and radial cracks occur in ductile [2] and brittle materials [3]. In the present work, we focus on brittle materials and we study the variation of the number of cracks with impact speed. Surprisingly, to our knowledge, this question has not been addressed before.

We perform controlled experiments of impacts on PMMA plates (Young's modulus $E = 3.3 \times 10^9$ Pa, density $\rho = 1190$ kg m^{-3} and Poisson ratio $\nu = 0.39$) with thickness h in the millimeter range ($h = 0.5$, 1.0, 1.5 and 3.0 mm) and side length of 15 cm. The plates are held to a square frame with magnets and we focus on the response at short times for which the boundary conditions do not affect the dynamics. A steel cylinder of mass $m = 16$ g with a hemispherical end of radius $R_i = 2.4$ mm is accelerated with a gas gun. It impacts the plates perpendicularly at their center at speeds in the range 10 to 120 m/s. The dynamics is recorded at 60000 frames per second. After impact, a transverse wave propagates on the plate. A pattern of radial cracks is observed early in the dynamics (Fig. 1). In general, the number of cracks is already chosen at the first observation ($t \leq 17$ μs after impact) and it does not change as the radial cracks extend. The angular distribution of the radial cracks is quite regular and at a given time all the cracks present approximately the same length. At later times, the transverse wave interacts with the boundaries of the plate and the pattern loses its regularity. As we increase the impactor speed from 10 m/s to 120 m/s the number of cracks increases from 3 to 11 (Fig 2 d).

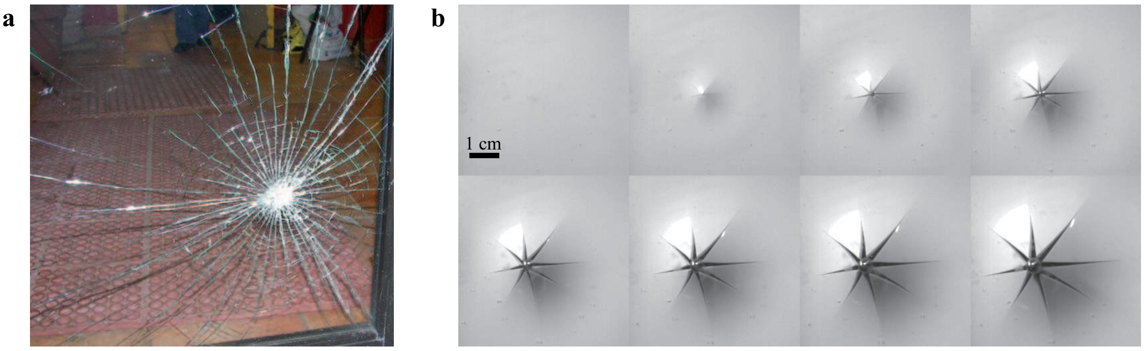

Figure 1. Star cracks pattern on impacted brittle plates. **a** A typical pattern of radial cracks on an impacted window. **b** A sequence of image showing the extension of 8 cracks after impact at speed $V = 56.7$ m/s on a PMMA plate of thickness $h = 1$ mm. The time interval between each frame is 33 μs.

SELECTION OF THE NUMBER OF CRACKS

To interpret the experiment, we abstract its crucial feature and build a model focussing on the effects of crack extension on the bending response of the plate. In particular, we neglect the effect of the early time dynamics in which 3D effects are important [4]. At time t after impact, we consider that bending is localized within an area of typical size $r_f = (cht)^{1/2}$ where $c = (E'/\rho)^{1/2}$ is the speed of sound with $E' = E/(1 - \nu^2)$. Neglecting the deceleration of the impactor, the displacement of the impact point is $w_0 = Vt$. To describe the shape of the impacted plate in the presence of radial cracks, we use a quasistatic model and consider a circular plate of external radius r_f with a central deflection w_0. The plate presents a pattern of n regularly spaced radial cracks extending up to radius $r_c = \xi r_f$. Using a simplified description of the geometry of the plate, we are able to calculate the shape and thus the elastic (bending) energy of the plate. Our model takes into account the non axisymmetric shape of a cracked plate and allows for the flattening of the petals (see also [5]). To determine crack extension and the optimal number of cracks, we rely on an approach inspired by Griffith description

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of fracture and we look for an optimal shape that minimizes the sum of elastic and surface energy

$$U_{tot} = \frac{\pi E' h^3 w_0^2}{3 r_f^2} k(\xi, n) + 2n\Gamma h \xi r_f = \frac{\pi E' h^3 w_0^2}{3 r_f^2} \left\{ k(\xi, n) + \frac{6}{\pi} \frac{\Gamma}{\rho h V^2} \left(\frac{h}{ct} \right)^{1/2} n\xi \right\} \tag{1}$$

The function k is the non-dimensional bending energy. It is equal to 2 when $\xi = 0$ (axisymmetric plate) and tends to 0.5 when $\xi \to 1$ and n large (limit of n beams). When ξ is close to 1, $k \sim 0.5 + b/n^2$. k decreases when ξ or n increase, showing that a plate with longer cracks and/or more cracks is less rigid. The total energy can be minimized to obtain the optimal ξ and n (Fig. 2 a).

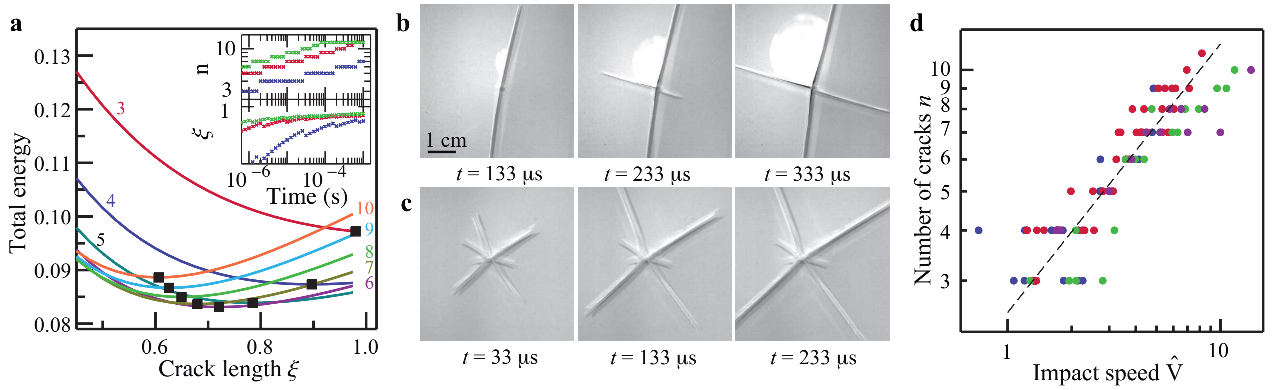

Figure 2. Selection of the pattern of cracks. **a** Total energy (non-dimensionalized by $(\pi/3)E'h^3(w_0/r_f)^2$) landscape for different values of n from 3 to 10 for impact speed $V = 60$ m/s at time $t = 5 \times 10^{-6}$ s for a plate of thickness $h = 1$ mm. All other parameters are given in the text. For each n a minimum can be found (black dots). The global minimum is obtained for $n = 6$. Inset: Optimal values of n and ξ as functions of time for three different impact speeds: $V = 20$ (blue), 60 (red) and 100 m/s (green). **b** Observation of increasing n for a low speed impact ($V = 15.7$ m/s) for a plate of thickness $h = 1.5$ mm. **c** For a thicker plate ($h = 3.0$ mm), numerous cracks are formed at impacts but only 4 open (here $V = 15.6$ m/s). **d** Evolution of the number of cracks with non-dimensional impact speed $\hat{V} = (\rho h V^2/\Gamma)^{1/2}$ (with $\Gamma = 200$ J/m^2) for plate thicknesses: $h = 0.5$ mm (blue), $h = 1.0$ mm (red), $h = 1.5$ mm (green), $h = 3.0$ mm (purple). The dashed line shows the $n \sim \hat{V}^{2/3}$ scaling law with an adjusted prefactor.

Our model predicts that the the optimal number of cracks slowly increases with t (typically $n^* \sim (ct/h)^{0.15}$) thus suggesting that new cracks may appear spontaneously as time grows. To determine the time at which the pattern is selected, we first mention that, our model being based on the bending response of the plate, it is not valid for $t < h/c$. At later times, it is important to consider the energy barrier between the states with different number of cracks [6]. Indeed our quasistatic model shows that the ratio of the available energy and energy barrier decreases as time grows. The times at which the energy barrier can no longer be crossed are typically of the order of h/c except for the smaller speeds. Thus for the sake of simplicity, we will assume that the pattern is selected at $t = h/c$. This argument shows that rearrangements should not occur during our observations except for small n as observed in our experiments (Fig. 2 b). We also mention that the pattern selection mechanism appear to be robust. In particular, for thicker plates ($h = 3.0$ mm) we observe situations as depicted on Fig. 2 c where even though several cracks are initiated only some of them extend.
Finally, a scaling law for the number of cracks can be deduced from our model. Taking the limit $\xi \to 1$, one can minimize U_{tot}, and we obtain a scaling law $n \sim (\rho h V^2/\Gamma)^{1/3} (ct/h)^{1/6}$. Taking a selection at $t = h/c$, one obtains

$$n \sim \left(\frac{\rho h V^2}{\Gamma} \right)^{1/3} \tag{2}$$

consistent with our experiments (Fig. 2 d).

References

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