ACOUSTIC METAMATERIALS FOR IMAGING BEYOND THE DIFFRACTION LIMIT

Xiaoming Zhou [a)] & Gengkai Hu
Department of Mechanics, School of Aerospace Engineering, Beijing Institute of Technology, Beijing, 100081, China

Summary Acoustic metamaterials with negative effective mass below a cut-off frequency are studied. The metamaterial model is a solid waveguide with clamped boundary conditions. Acoustic superlens with anisotropic effective mass is designed to create images beyond the diffraction limit. There are two different mechanisms with which evanescent fields can be maintained. One mechanism is based on the zero effective mass, and the other is based on the resonant tunneling. The metamaterial lens of anisotropic mass is designed, by which the imaging performance of the proposed lens with the spatial resolution beyond the diffraction limit is validated.

INTRODUCTION

Acoustic metamaterials are artificial composite materials with anomalous macroscopic material properties. Dynamic effective mass of metamaterials could become negative at frequencies where the building units resonate. A typical metamaterial possessing such behaviour is the composite with rubber-coated lead spheres embedded in the epoxy matrix [1]. Their effective mass is negative in a narrow band around a resonant frequency, and the underlying physics of negative effective mass has been discovered by a mass-spring structure [2]. In this work, the phenomenon of negative effective mass occurring below a cut-off frequency will be discussed based on both a mass-spring model and continuum material model. As an example of applications, such kind of metamaterials is used to design a superlens for acoustic imaging with the spatial resolution beyond the diffraction limit. The mechanism of the interaction between evanescent waves and the superlens is analyzed.

ACOUSTIC METAMATERIALS

In the classical mass-spring structure [2], negative effective mass below a cut-off frequency takes place when the inner mass is fixed. The equivalent mass-spring structure is shown in Fig. 1. The dynamic effective mass follows $m_{\text{eff}} = m_0\left(1-\frac{\omega_0^2}{\omega^2}\right)$, where $\omega_0=\sqrt{2G/m_0}$ is the cut-off frequency. The corresponding continuum material model is a rectangular solid waveguide with clamped boundary conditions. As a demonstration, Consider a two-dimensional waveguide filled by an elastic material and infinitely extended in the out-of-plane direction. When the waveguide has clamped boundary conditions, the dispersion relations are expressed by [3]

$$\omega^2/\omega_d^2 = m^2+\zeta^2, m=1,2,\ldots \tag{1a}$$

for P waves, and

$$\omega^2/\omega_s^2 = n^2+\zeta^2, n=1,2,\ldots \tag{1b}$$

for *SV* waves, where $\zeta=qh/\pi$, $\omega_d=\pi v_d/h$, $\omega_s=\pi v_s/h$, h is the width of the waveguide, v_d and v_s are respectively the wave velocities of longitudinal and shear waves, q is the propagation constant along the wave-guiding direction. For conventional elastic materials, the shear wave velocity is always less than longitudinal wave velocity, it means $\omega_s<\omega_d$. Therefore the dispersion relation (1) gives a lowest cut-off frequency ω_s for such waveguide, below which both *P* and *SV* waves are not allowed since the non-dimensional propagation constant ζ will be purely imaginary. The bandgap effect below the cut-off frequency ω_s can be attributed to negative effective mass. Consider the general dispersion relation $(q/\omega)^2=\rho_{\text{eff}}/\mu$, where μ is the shear modulus of the filling material. Let n=1 in Eq. (1b), effective mass density ρ_{eff} for the waveguide with clamped boundary condition is derived as

$$\rho_{\text{eff}}=\rho(1-\omega_s^2/\omega^2) \tag{2}$$

where ρ is the mass density of the filling material.

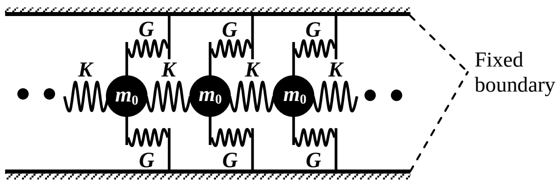

Fig. 1 The mass-spring model of metamaterials with negative effective mass below a cut-off frequency

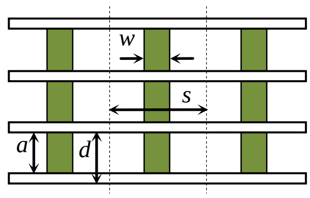

Fig. 2 The model of the superlens

a) Corresponding author. Email: zhxming@bit.edu.cn.

ACOUSTIC IMAGING BEYOND THE DIFFRACTION LIMIT

Conventional imaging systems capture propagating waves to produce images, while the sub-wavelength information of the objects can't be collected at the image plane because the evanescent waves carrying object's fine features decay rapidly in the near-field. The loss of evanescent wave components in the scattering fields is the fundamental reason of the diffraction limit of the conventional imaging system. Acoustic metamaterials with anisotropic mass provide a way to overcome the diffraction limit. Zhu et al designed the superlens made of the hole-drilled brass block to obtain the spatial resolution $\lambda/50$ (λ is the operating wavelength) [4], far beyond the diffraction limit $\lambda/2$. The dispersion curve of such anisotropic metamaterial is flat, meaning that the propagating wave vector is weakly dependent on the spatial frequency of evanescent waves. When the lens's thickness satisfies the Fabry-Pérot resonant condition, evanescent waves achieve the complete transmission.

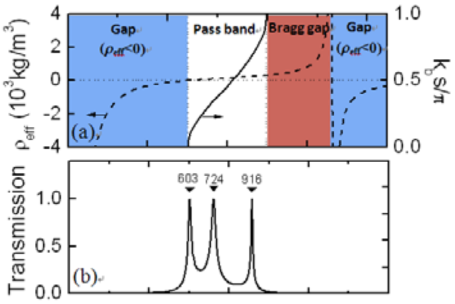

Fig. 3 (a)Effective mass ρ_{eff} and Bloch wave vector k_b versus frequencies evaluated in the unit cell of the proposed lens and (b) the transmission coefficients of the lens with three units

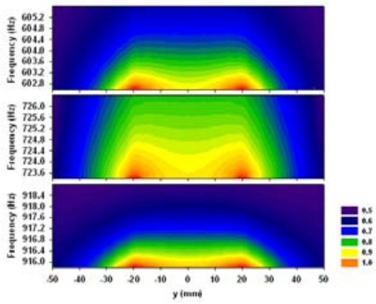

Fig. 4 Contour plots of pressure amplitude distributions in the image plane at frequencies around 603Hz, 724Hz and 916Hz.

By use of the anisotropic mass, we propose two different mechanisms for the extraordinary transmission of evanescent waves based on respectively the zero effective mass [5] and the resonant tunneling. Analyses on the transfer function of evanescent wave propagation in an anisotropic homogeneous material reveals that evanescent waves are prevent from decaying at zero effective mass. In physics, it can be explained by the mass law that high transmission is achieved in a material with the thickness as acoustically thin as possible. As the other different mechanism, the resonant tunneling occurs at large positive mass. The tunneling conditions are derived to be the lens thickness equal to the integer number of half wavelength of the periodic Bloch waves. We propose the model of the superlens to demonstrate the idea, as shown in Fig. 2. The designed lens consists of solid slabs with a periodic array of slits partially filled by elastic layers. The solid slabs are all fixed, so that effective mass of the lens in the vertical direction is infinite. In the horizontal direction, clamped elastic layers inside the slits undergo resonances at their lowest eigenfrequency. Effective mass follows the expression (2), the conditions for zero effective mass and resonant tunneling can be satisfied.

Fig. 3(a) shows effective mass (dashed line) and dispersion curve (solid line) computed for one unit cell. The transmission due to zero effective mass and the resonant tunneling effects will be examined in the pass band where effective mass is positive. Figure 3(b) shows the normal transmission coefficients of the lens with three units. In the pass band, three distinct transmission peaks are observed at 603Hz, 724Hz, and 916Hz. The first peak is the extraordinary transmission due to near-zero effective mass, and the latter two are induced respectively by the resonant tunneling of the odd and even modes. To verify the imaging effect, two monopole line sources separated by 40 mm are placed in front of the designed lens at the distance 1 mm and the image plane is taken 1 mm behind the lens. Figure 4 shows the contour plots of pressure amplitude distributions in the image plane at frequencies around 603Hz, 724Hz and 916Hz. It can be seen that the distance between two sources, which is less than the half wavelength, can be clearly distinguished from the images. The results demonstrate that the designed lens can create the images with spatial resolution beyond the diffraction limit at frequencies of zero effective mass and resonant tunneling.

CONCLUSIONS

Acoustic metamaterials with negative effective mass below a cut-off frequency are studied. Based on zero effective mass and resonant tunnelling effects, the metamaterial superlens is designed to create images beyond the diffraction limit.

References

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