ANALYSIS OF WAVE PROPAGATION BASED ON RADIAL BASIS COLLOCATION METHOD

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Summary A novel numerical method based on radial basis functions and collocation method is proposed for wave propagation. Standard collocation and weighted boundary collocation approaches yield significant errors in wave problems. Therefore, a new method can correct the inaccuracy in the solutions and the errors accumulated in the time integration is developed, and both explicit integration scheme and implicit integration scheme are derived. This method can be easily applied for low and high dimensional wave problems. Furthermore, the stability conditions are obtained and the relationships between control parameters and stability are evaluated. Eigenvalue analysis reveals the balance for choosing shape parameter and the number of collocation points. Numerical examples are simulated to examine and validated the proposed method.

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Radial basis collocation method which is based on strong form collocation with infinitely smooth radial basis functions has been applied for many boundary value problems because of its easy implementations and spectral convergence (Madych and Nelson [1]). However, the standard strong form collocation equations which are usually solved by least squares method introduce some errors on the boundary solutions. Weighted radial basis collocation method (Hu et al. [2]) can improve the accuracy on the boundaries in the static problems. While for time-dependent problems, both standard collocation and weighted boundary collocation approaches yield significant errors. Although some schemes such as statically condensation (Wang et al. [3]) were proposed to handle the solution difficulties in the dynamical problems, this method can only be applied for one-dimensional problem and hardly be used for high dimensional time-dependent problems.

In this paper, a novel numerical method based on radial basis functions and collocation method is proposed for wave propagation. The method first discrete the equations in the temporal domain, and then discrete the semi-discretized equation in the spatial domain. Based on such discretization scheme, the boundary conditions can be applied in each time step which grantees the accuracy on the boundaries in every time step and prevents the errors in the solutions. The simplified procedures are shown in Figure 1 where u defines the displacement, $\dot{u}$ is the velocity and n denotes the nth time step. Both explicit integration and implicit integration schemes are derived. This method can be easily applied for low and high dimensional wave problems.

The stability conditions are obtained and the relationships between control parameters and stability are evaluated. Results show that the shape parameter of the radial basis functions not only has strong influence on the dispersion errors, it also has profound influence on temporal stability conditions. Further, stability analysis shows that, in general, a larger critical time step can be used in RBCM with central difference temporal discretization than that for finite elements with the same temporal discretization.

Eignenvalue studies show that when shape parameter is relatively small, all the eigenvalues have negative real parts, and when shape parameter is relatively big, increasing the number of collocation points will bring the positive real parts to the corresponding eigenvalues. With big shape parameters, some eigenvlues with positive real part will be introduced which prevents the use of radial basis collocation method for wave problems. The range of suitable shape parameters is detailedly studied. Analysis also displays that when the number of collocation points is relatively small, all the eigenvalues have negative real parts even the shape parameter is very big. With the fixed number of collocation points, increasing the shape parameter will also bring the positive real parts to the corresponding eigenvalues. Therefore, there is a balance between choosing shape parameter c and the number of collocation points. Numerical examples are simulated to examine and validate the proposed method. The scheme reported herein can also be applied to other time-dependent problems.

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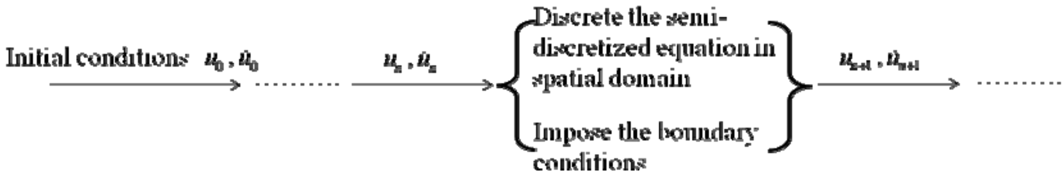

Figure 1. The procedures of the proposed scheme

References

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