SHOCK WAVE PROPAGATION IN FUNCTIONALLY GRADED CELLULAR STRUCTURES UNDER HIGH-VELOCITY IMPACT

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Summary Functionally graded cellular structures (FGCSs) with a continuous density-gradient distribution are constructed by using a novel method with changing cell-size distribution. Their dynamic responses are investigated by finite element method. Deformation localization and strength enhancement observed in uniform cellular structures are found also in FGCSs when subjected to a high-velocity impact. The one-dimensional continuum-based shock model is extended for FGCSs by employing a rate-independent, rigid–nonlinear hardening plastic idealization to model the material constitutive related to the local relative density. Predictions obtained from the shock model are compared well with the results of the cell-based models. The mechanism of shock wave propagation has well explained the responses of FGCSs under high-velocity impact, which may help to deduce the design principle of FGCSs.

INTRODUCTION

Functionally graded cellular structures (FGCSs) compared to uniform cellular structures (UCSs) may exhibit superior mechanical properties such as impact resistance and energy absorption [1-2]. On one hand, some types of FGCSs have been proposed in the literature [2], but they are limited to stepwise, discontinuous density-gradient distribution formed by joining various uniform-density layers. On the other hand, stress wave propagation in FGCSs under impact is too complex to understand, and so the design principle of FGCSs is still not very clear. The present study aims to present a new type of FGCSs with a continuous density-gradient distribution and obtain some understanding of deformation and stress wave propagation mechanisms of FGCSs under high-velocity impact.

FINITE ELEMENT MODELS

We consider a type of two-dimensional FGCSs with a linear distribution of relative density along the X axis, written as

$$\rho(X) = \rho_{\text{avg}}[1+\lambda(X/L-1/2)] \tag{1}$$

where λ is a dimensionless density-gradient parameter and ρ_{avg} the average relative density of specimen with length L, width W and thickness H. A novel method with changing cell-size distribution but keeping the cell-wall thickness identical is developed to change the relative density distribution of FGCS specimens. The 2D Voronoi technique [3] with a new principle of seeding nuclei is used to generate FGCS specimens. Nuclei are randomly seeded by the principle that the distance between any two nuclei i and j is not less than the minimum allowable distance

$$\delta_{ij}^{\min} = (1-k)2h/\rho((X_i+X_j)/2)\,, \tag{2}$$

where h is the cell-wall thickness, k the cell irregularity, and (X_i, Y_i) the location of nucleus i. In this study, we take $\rho_{\text{avg}} = 0.1$, $k = 0.2$, $h = 0.2625$ mm, $L = 400$ mm, $W = 100$ mm and $H = 1$ mm.

Three FGCS samples are considered, i.e. $\lambda = 0$ (a UCS) and $\lambda = \pm 1$. A FGCS specimen with $\lambda = -1$ is represented in Fig. 1a. Constant-velocity impact tests of FGCS specimens are performed by using finite element (FE) method with ABAQUS/ Explicit code. The specimens are put between two rigid platens, as schematically presented in Fig. 1a. The right platen is stationary and the left platen moves rightward with a constant velocity, V_0. The cell-wall material of FGCSs is taken to be elastic-perfectly plastic with the Young's modulus, yield stress, Possion's ratio and mass density being $E = 69$ GPa, $\sigma_y = 170$ MPa, $\nu = 0.3$ and $\rho_s = 2700$ kg/m^3, respectively.

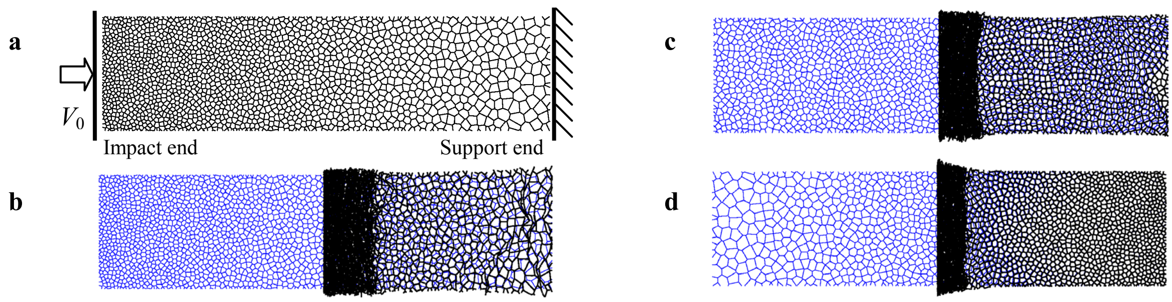

Fig. 1. (a) The finite element model of a FGCS specimen with $\lambda = -1$ under constant-velocity impact. The deformation patterns of FGCS specimens with (b) $\lambda = -1$, (c) $\lambda = 0$ and (d) $\lambda = 1$ at time $t = 2$ ms when the impact velocity is 100 m/s.

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RESULTS AND DISCUSSION

Deformation localization and strength enhancement observed in UCSs [3] are found also in FGCSs with $\lambda \neq 0$ under high-velocity impact, see Figs. 1 and 2. Deformation patterns in Fig. 1 show that cells collapse layer-by-layer and a deformation front propagates through the specimens. In Fig. 2, the impact stress is much higher than the support stress during impact before densification. The magnitude of impact stress of the UCS specimen ($\lambda = 0$, Fig. 2b) remains at a constant level before densification, but that of the FGCS specimen with $\lambda = -1$ ($\lambda = 1$) has a significant decrease (increase) tendency, Figs. 2a and c. The front travels faster than the moving platen due to the accumulation of crushed cells. The velocity across the front is apparently discontinuous, which is explained with structural shock propagation [4]. A one-dimensional continuum-based shock model [4] has been developed for UCSs to predict the strength enhancement. Here, we will extend this model for FGCSs under high-velocity impact.

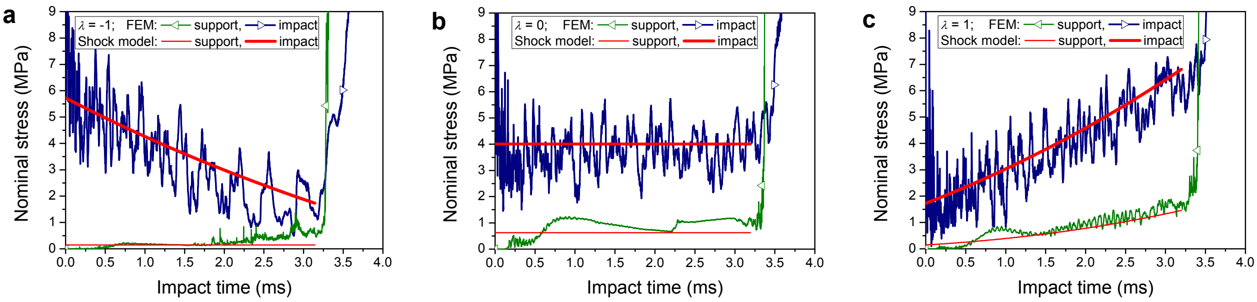

Fig. 2. Nominal stresses at the support and impact ends of FGCS specimens with (a) $\lambda = -1$, (b) $\lambda = 0$ and (c) $\lambda = 1$. Here, $V_0 = 100$ m/s.

The material constitutive of a FGCS depends on the local relative density, which is modeled as a rate-independent, rigid–nonlinear hardening plastic idealization with the quasi-static stress–strain relation in the form [5]

$$\sigma(\varepsilon) = \sigma_0 + \gamma\,\varepsilon/(1-\rho) - \alpha\ln\{1-[\varepsilon/(1-\rho)]^{\beta}\}\,, \tag{3}$$

where σ_0, α, β and γ are coefficients. Data fitting of the quasi-static nominal stress–strain curves of UCSs with relative density ranging from 0.4 to 1.6 with a step of 0.1 gives $\sigma_0/\sigma_y = 0.4441\rho^{2.080}$, $\gamma/\sigma_y = 0.1078\rho^{1.478}$, $\alpha/\sigma_y = 0.04712\rho^{0.2392}$ and $\beta = 1.025\rho^{-0.8015}$. Consider a plastic shock front propagates along the X axis with Lagrangian speed $\dot{\Phi}(t)$, where $\Phi(t)$ is the Lagrangian position of the front at time t and the dot symbol (.) denotes the derivative with respect to the time. The stresses at the impact and support ends of specimen are equal to those behind and ahead of the front, respectively. Based on the one-dimensional stress wave theory, the stress behind the front is determined as

$$\sigma_B = \sigma(\varepsilon_B) = \min_{\Phi\le X\le L}\{\sigma_0(\rho(X))\} + \rho(\Phi)\,\rho_s V_0^2/\varepsilon_B\,, \tag{4}$$

where ε_B is the strain behind the front. The last term in Eq. (4) is due to shock enhancement. Given the location of the front Φ, the strain behind the front ε_B can be determined from Eqs. (3) and (4) with Newton's iteration method. The kinematic compatibility condition across the front gives $\dot{\Phi}(t) = V_0/\varepsilon_B$, which leads to

$$t = \int_0^{\Phi}\frac{1}{\dot{\Phi}}\,d\Phi = \frac{1}{V_0}\int_0^{\Phi}\varepsilon_B\,d\Phi\,. \tag{5}$$

The predictions from this extended shock model are shown in Fig. 2, which are compared well with those of cell-based FE models. This shock model may help to design the density-gradient distribution of FGCSs for a desired response.

CONCLUSIONS

Dynamic responses of FGCSs are investigated by using cell-based FE models and a continuum-based shock model. A novel method with changing cell-size distribution is developed to construct FGCS specimens. Deformation localization and strength enhancement observed in UCSs are found also in FGCSs when the impact velocity is high. Predictions obtained from the extended shock model are compared well with the results of the cell-based FE models. The mechanism of shock wave propagation has well explained the responses of FGCSs under high-velocity impact.

References

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