# Dynamic Response Analysis of Plane Composite Trusses with the Method of Reverberation Ray Matrix

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## Introduction

Fiber reinforced laminated composite are widely used as structural materials in many industry departments, such as aerospace, aviation, atomic nucleus, automobile, etc. Due to certain advantages over conventional structural materials. Apparently, dynamics response analysis of composite structures is of practical importance in many engineering applications. The Method of Reverberation Ray Matrix (MRRM) has been developed by Pao *et al.*[1] recently based on the theory of wave propagation for transient analysis of truss or frame type structures. The singularity problem, which also arises in other frequency domain matrix methods, is solved in terms of Neumann series expansion. The method has been demonstrated accurate in transient response of a lab truss by experiment.

There are very a few literatures available which consider the impact response and dynamics analysis of laminated composite beams by different method. However, there are very few articles about the dynamic analysis of laminated composite truss.

In this study, the dynamics response of laminated composite beams under an impact loading with MRRM based on Bernoulli-Euler theory were analyzed. The dynamic response of the trusses composed of the same beams is solved.

## Analysis

With the Bernoulli-Euler theory, the governing equations for the laminated beam in frequency domain are:

$$\hat{u}+\frac{A_{11}}{\rho A\omega^2}\hat{u}_{,xx}-\frac{B_{11}}{\rho A\omega^2}\hat{w}_{,xxx}=0$$
$$\hat{w}+\frac{B_{11}}{\rho A\omega^2}\hat{u}_{,xx}-\frac{D_{11}}{\rho A\omega^2}\hat{w}_{,xxx}=0 \qquad (1)$$

where $A_{ij}$, $B_{ij}$, $D_{ij}$ are the stiffness coefficients. The axial force, shear force and bending moment are:

$$\hat{F}_N = A_{11}\frac{\partial \hat{u}}{\partial x}-B_{11}\frac{\partial^2 \hat{w}}{\partial x^2}$$
$$\hat{F}_S = B_{11}\frac{\partial^2 \hat{u}}{\partial x^2}-D_{11}\frac{\partial^3 \hat{w}}{\partial x^3} \qquad (2)$$
$$\hat{M} = -B_{11}\frac{\partial \hat{u}}{\partial x}+D_{11}\frac{\partial^2 \hat{w}}{\partial x^2}$$

The axial, lateral displacement u, w and angle of rotation ϕ are assumed to be

$$\begin{Bmatrix}\hat{u}^{JK}(x,\omega)\\ \hat{w}^{JK}(x,\omega)\\ \hat{\phi}^{JK}(x,\omega)\end{Bmatrix}=\begin{bmatrix}1 & 1 & \beta_2^{JK} & -\beta_2^{JK} & \beta_3^{JK} & -\beta_3^{JK}\\ \beta_1^{JK} & -\beta_1^{JK} & 1 & 1 & 1 & 1\\ -ik_1\beta_1^{JK} & -ik_1\beta_1^{JK} & -ik_2^{JK} & ik_2^{JK} & -ik_3^{JK} & ik_3^{JK}\end{bmatrix}\begin{Bmatrix}d_1^{JK}e^{-ik_1^{JK}x}\\ a_1^{JK}e^{ik_1^{JK}x}\\ d_2^{JK}e^{-ik_2^{JK}x}\\ a_2^{JK}e^{ik_2^{JK}x}\\ d_3^{JK}e^{-ik_3^{JK}x}\\ a_3^{JK}e^{ik_3^{JK}x}\end{Bmatrix} \qquad (3)$$

where $k_i$(i=1,2,3) is wave number, $d_i$ and $a_i$ (i=1,2,3) are amplitudes of departure wave and arriving wave respectively. Coefficients and $\beta_i$ (i=1-3) could be found in Ref.[2]. A 17-bar truss as shown in Fig.1 is used as an example.

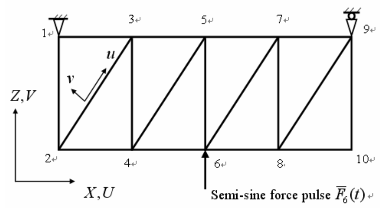


Fig.1. The truss and the coordinate systems

If not considering the concentrated mass in the joint $J$, equation of motion of joint $J$ can be written as[2]

$$\sum_{K=1}^{n^J}[\hat{F}_N^{JK}(0,\omega)\cos\phi^{JK}-\hat{F}_S^{JK}(0,\omega)\sin\phi^{JK}]+\hat{f}_X^J=0$$
$$\sum_{K=1}^{n^J}[\hat{F}_N^{JK}(0,\omega)\cos\phi^{JK}+\hat{F}_S^{JK}(0,\omega)\sin\phi^{JK}]+\hat{f}_Z^J=0 \qquad (4)$$
$$\sum_{K=1}^{n^J}\hat{M}^{JK}(0,\omega)+\hat{m}^J=0$$

The displacements compatibility equations at each joint are:

$$\hat{u}^{JK}(0,\omega)\cos\phi^{JK}-\hat{w}^{JK}(0,\omega)\sin\phi^{JK}=\hat{U}^J(\omega)$$
$$\hat{u}^{JK}(0,\omega)\sin\phi^{JK}+\hat{w}^{JK}(0,\omega)\cos\phi^{JK}=\hat{W}^J(\omega) \qquad (5)$$
$$\hat{\phi}^{JK}(0,\omega)=\hat{\Phi}^J(\omega)$$

Substituting Eq.(2) and Eq.(3) into Eq.(4) and Eq.(5) the following relation is obtained:

$$\begin{bmatrix} \boldsymbol{D}_{11} & \boldsymbol{D}_{12} \\ \boldsymbol{D}_{21} & \boldsymbol{D}_{22} \end{bmatrix}^J \begin{Bmatrix} \boldsymbol{d}^J \\ \boldsymbol{H}^J \end{Bmatrix} = \begin{bmatrix} \boldsymbol{A}_1 \\ \boldsymbol{A}_2 \end{bmatrix}^J \{\boldsymbol{a}^J\} - \begin{Bmatrix} 0 \\ \boldsymbol{f}^J \end{Bmatrix} \tag{6}$$

or in simplified form:

$$\{d\}^J = [S]^J \{a\}^J + \{s\}^J \tag{7}$$

where $[S]^J$ is the scattering matrix and $\{s\}^J$ is load vector. It is shown by Pao[1] that $\{a\}$ is related to $\{d\}^J$ with $\{a\}^J=[P]^J[U]^J\{d\}^J$ where $[P]^J$ and $[U]^J$ are phase matrix and permutation matrix, respectively. So that $\{d\}^J$ can be expressed as

$$\{d\}^J=[I-R]^{-1}\{s\}^J \tag{8}$$

where $[R]=[S][P][U]$ is the Reverberation matrix. Once $\{d\}^J$ and $\{a\}^J$ are solved, the displacement, force and moment can be calculated from Eqs.(3) and (2).

## Results

The lamina properties are $E_1$=144GPa, $E_2$=9.65GPa, $G_{23}$=3.45GPa, $G_{12}=G_{13}$=4.14GPa, $\nu_{12}$=0.3, $\rho$=1389kg/m$^3$. The beam length L=1m with cross section 1cm×1cm. A smoothed triangular force with amplitude 4.4N is applied at the right tip. Figure 2 shows comparison of MRRM with the result of FEM[3] and SEM[4].

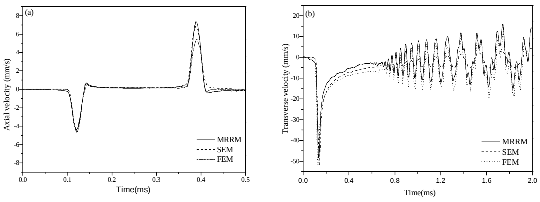


Fig.2. Comparison of (a) Axial velocity response due to axial tip load; (b) Transverse velocity response due to transverse tip load of laminated cantilever beam by MRRM, FEM(Ref.[3]) and SEM(Ref.[4])

Then three ply-stacking sequences (0), ($0_5/45_3/60_2$), and ($0_5/90_5$) are used which yield a coupling factor $r=\sqrt{B_{11}/(A_{11}D_{11})}$ = 0, 0.135, 0.314 to examine the influence of asymmetric lamination. A half sine pulse load with amplitude 4.4N is applied at the right tip. In Figure 3, (a) axial tip velocity due to axial load, (b) transverse velocity due to axial load and (c) transverse velocity due to transverse load are presented.

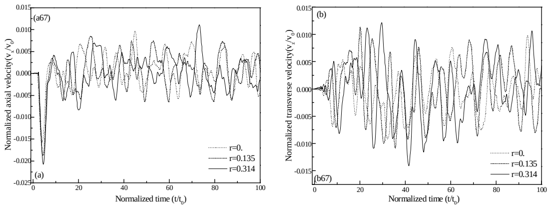


Fig.3. (a)Axial velocity response (b)Transverse velocity response at the midpoint of member 67, subject to half-sine force pulse at the joint 6.

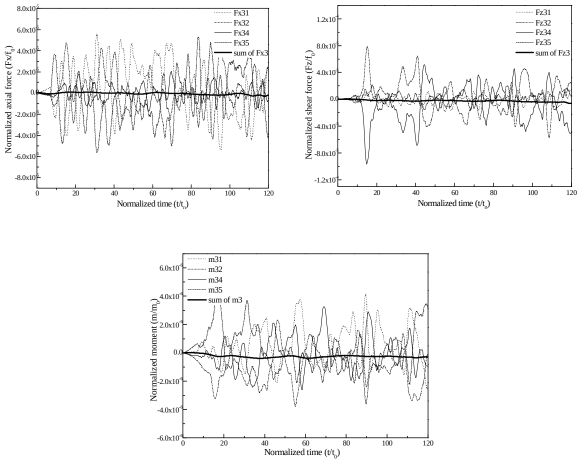


Fig.4. Equilibrium of forces and moment at joint 3.

## Conclusion

The reverberation ray matrix method, which was developed initially for transient response of laminated composite truss type structures, The reliability and accuracy of MRRM is verified by comparing with SEM and FEM solution to cantilever laminated beam under sudden smoothed triangular pulse. It is found that with the increase of axial-bending coupling factor, the amplitude of axial and transverse velocity due to axial load increases while the transverse velocity due to transverse load does not differ much. Thus it can be seen, MRRM can be used as analyze the dynamic response of laminated composite truss under sudden force pulse.

## Reference

1. Y. H. Pao, D. C. Keh, S. M. Howard, Dynamic Response and Wave Propagation in Plane Trusses and Frames. AIAA Journal, 1999, **37**(5) 594-603.
2. F. X. Miao, A study on the vibration and transient responses of frames by the method of reverberation ray matrix. PhD dissertation, Shanghai Jiao Tong University, 2009.
3. A. Chakraborty, D.R. Mahapatra, S. Gopalakrishnan, Finite element analysis of free vibration and wave propagation in asymmetric composite beams with structural discontinuities. Composite Structures, 2002, 55: 23-26.
4. D.Roy Mahapatra and S. Gopalakrishnan, Spectral-element-based solutions for wave propagation analysis of multiply connected unsymmetric laminated composite beams, Journal of Sound and Vibration. 2000, 237(5): 819-836.