NUMERICAL MODELING OF LASER GENERATION OF ULTRASOUND BY COUPLED THERMOELASTICITY

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Summary Laser generation of ultrasound is investigated by numerical techniques in the presented paper. The coupled heat conduction and wave equations are solved with finite differences using a combination of implicit and explicit integration techniques. Wave fields generated by pulsed laser irradiation are presented for various pulse durations in the ps-ns range and the amplitude of the generated longitudinal bulk wave is investigated with the model as a function of the pulse duration at constant energy.

INTRODUCTION

Laser generation of ultrasound has been widely and successfully used in experimental acoustics in the past decades since it enables contactless excitation of bulk, surface or guided elastic waves up to the GHz range. The modeling of the thermoelastic generation mechanism is of particular interest. Thereby, the generation of the elastic waves is described by the coupled heat and wave equations [1, 2, 3]. Point-source representation of the thermal expansion has been introduced as the surface center of expansion in [1] using a double integral transform technique. Similar solutions have been pursued by later authors and extended to line sources [2]. For such analytic solutions, however, certain simplifications are required, such as a step-like temperature rise with a point or line spatial distribution for the surface heating. For general cases with more realistic distributions of the heat flux the inverse integral transforms remain a great challenge and consequently numerical inversion techniques were applied [3]. Numerical solutions such as finite elements or finite differences have been rarely pursued, though they offer a greater flexibility in their applications [4, 5]. Here, a combination of an implicit-explicit finite difference technique will be introduced considering also the thermal feedback by mechanical deformation.

GOVERNING EQUATIONS AND THEIR DISCRETISATION BY FINITE DIFFERENCES

The governing equations of thermoelasticity in the linear elastic case are given by the coupled heat conduction and wave equations describing the coupling of the displacement and temperature fields. In the presented work the case the generated wave field is investigated for a line-focused laser source, which is approximated as a 2D case with plane-strain boundary conditions [3]. Using the Einstein summation convention the governing equations are given as:

$$kT_{,ii} = \rho c_v (T_{,t} + \tau T_{,tt}) + \alpha\beta T_0 (\varepsilon_{kk,t} + \tau\varepsilon_{kk,tt}) - q \tag{1}$$

$$\rho u_{i,tt} = (\lambda + \mu) u_{j,ij} + \mu u_{i,jj} - \alpha\beta T_{,i}\,, \quad \text{with:} \quad i,j,k = 1,2 \tag{2}$$

where u denotes the displacements, T the temperatures, k the thermal conductivity, c_v the specific heat of the material at constant deformation, ρ the density, α the thermal expansion coefficient, β the bulk modulus, τ the material relaxation time, T_0 the reference temperature, q the external heat flux, ε the strains and λ, μ being the Lame parameters. The hyperbolic heat conduction equation assumes a finite heat propagation speed, which is approximated as five times the longitudinal wave velocity c_l leading to a thermal relaxation time of $\tau = k(25\rho c_v c_l^2)^{-1}$. Equations (1)-(2) are discretised in the space domain using the finite differences technique with staggered grids. Hence, the grids of the displacements and stresses are shifted relative to each other by a half grid dimension (see [6]) and the grid of the temperatures is identical with the grid of the normal stresses. At the boundaries stress free boundary conditions are applied to the wave equations and adiabatic boundary condition to the heat conduction equation. Moreover, due to symmetry only one half of the area is modeled using symmetric boundary conditions. The wave equation is efficiently discretised by the conditionally stable, explicit Euler-method [6] whereby the unknown displacements in the $(n+1)th$ time step are explicitly expressed from the displacements ($\mathbf{u}$) and stresses ($\boldsymbol{\sigma}$) in the previous time steps ($(.)^n, (.)^{n-1}$), i.e., with independent equations:

$$\mathbf{u}_x^{n+1} = 2\mathbf{u}_x^n - \mathbf{u}_x^{n-1} + \tfrac{\Delta t^2}{\rho\Delta x}\delta_x^1 \boldsymbol{\sigma}_{xx}^n + \tfrac{\Delta t^2}{\rho\Delta y}\delta_y^1 \boldsymbol{\sigma}_{xy}^n, \qquad \mathbf{u}_y^{n+1} = 2\mathbf{u}_y^n - \mathbf{u}_y^{n-1} + \tfrac{\Delta t^2}{\rho\Delta y}\delta_y^1 \boldsymbol{\sigma}_{yy}^n + \tfrac{\Delta t^2}{\rho\Delta x}\delta_x^1 \boldsymbol{\sigma}_{xy}^n \tag{3}$$

where δ_x^1, δ_y^1 are the first order finite differential operators in x and y directions and $\Delta t, \Delta x, \Delta y$ are the temporal and spatial grid dimensions. The heat conduction equation is given in matrix form after spatial discretisation as:

$$\tau\ddot{\mathbf{T}} + \dot{\mathbf{T}} + \mathbf{B}\mathbf{T} = \mathbf{q} - \tfrac{\alpha\beta T_0}{\rho c_v}(\dot{\boldsymbol{\epsilon}}_{xx} + \dot{\boldsymbol{\epsilon}}_{yy} + \tau\ddot{\boldsymbol{\epsilon}}_{xx} + \tau\ddot{\boldsymbol{\epsilon}}_{yy}), \tag{4}$$

where $\boldsymbol{\epsilon}_{xx}, \boldsymbol{\epsilon}_{yy}$ denotes the x and y components of the strains, respectively. Equation (4) is integrated implicitly by the unconditionally stable Wilson Θ method with the same time step as used in the wave equation. Due to the implicit integration the coefficient matrix $\mathbf{B}$ must be inverted, however, the number of the coupled linear equations is only one-third of the total degrees of freedom of the system (DOF). A similar combination of explicit and implicit integration techniques has been applied in [5] in 1D, however, neglecting the thermal feedback and the hyperbolic term.

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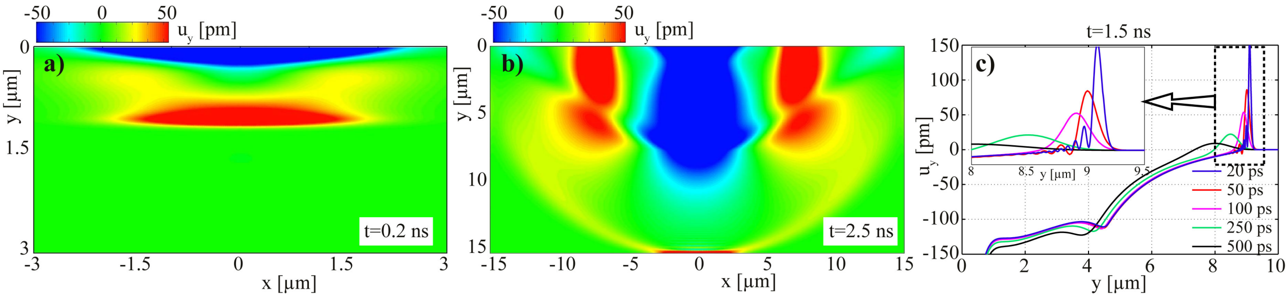

Figure 1. Snapshots of the vertical displacements u_y at t =0.2 ns and t =2.5 ns after the excitation with a 50 ps laser pulse. The short light pulse excites first a high frequency bulk wave (a) and later also other wave types such as the Rayleigh wave become visible (b). c): variation of the epicentral wave form for different pulse durations at constant energy (t =1.5 ns).

Numerical results - generation of bulk and surface waves

In the following the generation of bulk and surface waves is investigated by the presented numerical model. In particular, a pulsed laser is considered with 0.05 mJ pulse energy focused to a line with a half width of 2 μm and length of 10 mm. An aluminum alloy is modeled with the material properties given in [3]; a reflectivity of 0.95 is assumed for the simulation. The laser source is assumed to exhibit a spatial-temporal Gaussian distribution modeled as surface flux [3]. Snapshots of the resulting vertical displacements u_y of the wave field are shown in Fig.1(a)-(b) for a pulse duration of 50 ps. Figure 1(a) shows the generated longitudinal bulk wave at t =0.2 ns after excitation propagating into the bulk material. This wave possesses a very high frequency content due to the short excitation pulse [5]. At t =2.5 ns after excitation [Fig.1(b)], however, also the generated Rayleigh wave becomes visible. It is apparent that the width of the Rayleigh wave ($\sim$3.5 μm) coincides well with the width of the laser source (4 μm); hence its frequency content depends upon this width. As a practical application of the model the amplitude of the generated longitudinal bulk waves is investigated by keeping the pulse energy constant and varying the pulse duration between 20 ps and 500 ps. In Fig.1(c) epicentral waveforms are shown at t =1.5 ns after excitation. The amplitude of the wave strongly decreases with increasing pulse duration while the pulse is prolonged, showing, that the generation of the bulk wave is more efficient with shorter pulses.

CONCLUSION

The coupled equations of thermoelasticity have been investigated in the presented paper. In particular a numerical model based on an implicit-explicit finite difference method has been introduced to describe the laser generation of ultrasonic waves. The model includes coupling between the two sets of equations in both directions through the thermal expansion and the thermal feedback by the mechanical stresses. Due to the combination of explicit and implicit methods the number of the coupled linear equations is, however, only one-third of the total degrees of freedom of the system. The model has been applied to investigate the generation process by pulsed laser sources with durations in the ps-ns range, presenting snapshots of the generated wave fields. In addition, the amplitude of the generated bulk longitudinal waves is investigated with the model. It has been shown, that this amplitude strongly depends upon the pulse duration and is increasing with decreasing pulse duration at constant energy. The presented results show the applicability and flexibility of numerical methods to solve and investigate such coupled problems.

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