SCATTERING OF SH-WAVE BY MULTIPLE CYLINDRICAL CAVITIES AND A LINEAR CRACK IN ELASTIC SEMI-SPACE

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Summary Cylindrical cavity exists widely in natural media, engineering materials and modern municipal construction, and defects are usually found around the cavity. In this paper, Green's function method is used to investigate the problem of dynamic stress concentration of multiple cavities and a linear crack in elastic semi-space for incident SH wave. The expressions of the displacement and stress are given, and an example is provided to show the effect of cavities and crack on the dynamic stress concentration.

INTRODUCTION

In natural medium, engineering materials and structures, it can be found that there are cavities everywhere. When structure is impacted by dynamic load, the scattering field will be produced because of the cavities, and it could cause dynamic stress concentration at the edge of the cavities. When the structure is overloaded or the load is changed regularly, cracks emerge and spread near the cavities. In theory of elastic wave motion, cavity and crack are two danger factor. Dynamic stress concentration could greatly decrease the bearing capacity of structure, and reduce the service life of structure.

In this paper, the method of Green's function is used to investigate the problem of dynamic stress concentration of multiple cylindrical cavities and a linear crack in elastic semi-space for incident SH wave[1]. Multi-polar coordinate system is used too. Firstly, a Green's function is constructed for the problem, which is a fundamental solution of displacement field for an elastic semi-space possessing multiple cylindrical cavities while bearing out-of-plane harmonic line source force at any point: Secondly, in terms of the solution of SH-wave's scattering by an elastic semi-space with multiple cylindrical cavities, anti-plane stresses which are the same in quantity but opposite in direction to those mentioned before, are loaded at the region where the crack is in existent actually; Finally, the expressions of the displacement and stresses are given for the problem. Then, by using the expressions, an example is studied to show the effect of crack on the dynamic stress concentration around cylindrical cavities.

MODEL AND GREEN'S FUNCTION

The model is shown as Fig.1, elastic semi-space containing multiple cylindrical cavities and a linear crack. In this paper, the anti-plane shear wave model is studied. Based on the complex function theory, the governing equation of W can be written as

$$\frac{\partial^2 W}{\partial z \partial \overline{z}} + \frac{1}{4} k^2 W = 0 \tag{1}$$

The Green's function used in this paper is regarded as the displacement response to the elastic semi-space containing multiple cylindrical cavities impacted by anti-plane harmonic linear source force at any point . The displacement in the elastic space is expressed as $G(z, \overline{z}, t)$, z_G stands for the position of the linear source force in complex plane. The boundary conditions can be expressed as below:

$$\tau_{\theta z} = 0 (where\ \theta = 0\ and\ \theta = \pi)\ ,\ \tau_{\theta z} = \delta(z - z_G)\ \ (z = z_G)\ ,\ \tau_{rz} = 0\ \ (|z - c_m| = R_m, m = 1, 2, ..., N) \tag{2}$$

The wave displacement due to the line source load $\delta(z - z_G)$ and the scattering wave can be written as:

$$G^{(i)} = \frac{i}{4\mu}[H_0^{(1)}(k|z - z_G|) + H_0^{(1)}(k|z - \overline{z}_G|)]e^{-i\omega t} \tag{3}$$

$$G^{(s)} = \sum_{j=1}^{N} \sum_{n=-\infty}^{\infty} A_n^j [H_n^{(1)}(k|z - c_j|)(\frac{z - c_j}{|z - c_j|})^n + H_n^{(1)}(k|z - \overline{c}_j|)(\frac{z - \overline{c}_j}{|z - \overline{c}_j|})^{-n}] \tag{4}$$

where A_n^j are unknown coefficients.

The wave field G must satisfy the stress free condition on cavities. So by using the method of transferred coordinate, when the origin of coordinate is c_m, G can be written as:

$$G = \frac{i}{4\mu}[H_0^{(1)}(k|z_m + c_m - z_G|) + H_0^{(1)}(k|z_m + c_m - \overline{z}_G|)] + \sum_{j=1}^{N} \sum_{n=-\infty}^{\infty} A_n^j [H_n^{(1)}(k|z_m - d_{mj}|)(\frac{z_m - d_{mj}}{|z_m - d_{mj}|})^n + H_n^{(1)}(k|z_m - d'_{mj}|)(\frac{z_m - d'_{mj}}{|z_m - d'_{mj}|})^{-n}] \tag{5}$$

where $d_{mj} = c_j - c_m$.

Substituting the wave filed G to the boundary conditions, it can be obtained that

$$\sum_{j=1}^{N} \sum_{n=-\infty}^{\infty} A_n^j \varphi_{mn}^j = \varphi_m \tag{6}$$

By multiplying both sides of (6) with $e^{-im\theta_m}$ and integrating in interval $[-\pi, \pi]$,it can be obtained that

$$\sum_{j=1}^{N}\sum_{n=-\infty}^{\infty} A_n^j \Psi_{mn}^j = \Psi_m \tag{7}$$

Equation (7) is a set of infinite algebraic equation for determining the coefficients A_n^j.
Substituting the coefficients A_n^j to Expression (5), the total wave field G of this problem can be obtained.

EXPRESSION OF DISPLACEMENT AND STRESS FOR THE MODEL

Firstly, we consider the incidence of SH-wave to the infinite linear elastic semi-space containing multiple cylindrical cavities. The incident displacement field $W^{(i)}$ harmonic to time can be written as follows[2]:

$$W^{(i)} = W_0 e^{\frac{ik}{2}(ze^{-i\alpha}+\bar{z}e^{i\alpha})} + W_0 e^{\frac{ik}{2}(ze^{i\alpha}+\bar{z}e^{-i\alpha})} \tag{8}$$

where α is the incident angle.
The scattering wave incited by cavities can be written as:

$$W^{(s)} = \sum_{j=1}^{N}\sum_{n=-\infty}^{\infty} B_n^j [H_n^{(1)}(k|z-c_j|)(\frac{z-c_j}{|z-c_j|})^n + H_n^{(1)}(k|z-\bar{c}_j|)(\frac{z-\bar{c}_j}{|z-\bar{c}_j|})^{-n}] \tag{9}$$

where B_n^j are unknown coefficients.
The boundary conditions can be expressed as below:

$$\tau_{rz}^{(i)} + \tau_{rz}^{(s)} = 0 \quad (|z-c_m| = R_m, m = 1,2,\ldots,N) \tag{10}$$

By using the same method which is used to confirm the unknown coefficients of the Green's function, the wave field $W^{(z)} = W^{(i)} + W^{(s)}$ can be obtained.
We consider the scattering problem of incident SH-wave when multiple cylindrical cavities and linear crack exist at the same time. the space is separated along the crack and a pair of anti-plane opposite forces with the multitude $-\tau_{\theta z}^{(z)}$ are applied to up and down section of the region where crack will appear, therefore the resultant force on up (or down) section of the region is zero, which can be thought as crack. Hence, the total displacement and stress field can be written as follows:

$$W = W^{(i)} + W^{(s)} - \int_{z_1}^{z_2} \tau_{\theta z}^{(z)}|_{z=z_0} \times G(z, z_G)dz_G \text{ , } \tau_{\theta z} = \tau_{\theta z}^{(z)} - \int_{z_1}^{z_2} \tau_{\theta z}^{(z)}(z_0) \cdot \tau_{\theta z}^{(G)}(z, z_G)dz_G \tag{11}$$

EXAMPLE

In this paper, we pay attention to a representative kind of models. There are two cavities. The radius of the cavities equals 1 , and the length of the crack is 2. The other parameters are shown in the figures.Fig.2 show the variation of dynamic stress concentration factor at the left cavity edge as the incident angle of SH-wave is 60°. From the figure , it can be shown that the interaction of multiple cylindrical cavities and a linear crack should not be neglected.

CONCLUSIONS

In this paper, by using the technique of crack-division, a new method is given to solve the interaction problem of multiple cylindrical cavities and a Linear Crack in elastic semi-space by incident SH-wave. By using the method an example is solved. The method in the paper could be used to study some other correlative problem.

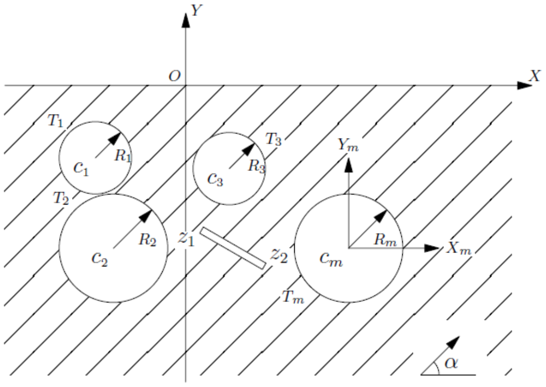

Fig.1. Model of the problem

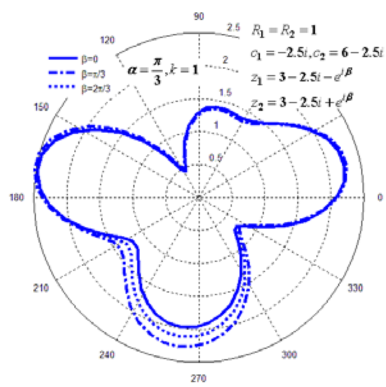

Fig.2. Influence of crack to DSCF at the left cavity edge

References

[1] H.L. Li, G.C. Han, H. Li : Interaction of Circular Lining and Interior Linear Crack. *Key Eng. Mate* **348-349**:521-524,2007.
[2] H.L. Li, C. Zhang: Green's Function Solution of the Semi-space with Double Shallow-buried Cavities. *Key Eng. Mate* **417-418**:145-148,2010.