

# GRANULAR ELEMENT METHOD FOR COMPUTATIONAL PARTICLE MECHANICS

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**Summary** By using parametric curves to represent grain geometries, an alternative approach to spherical- and polyhedra- based discrete element methods is proposed whereby the need for complicated and ad-hoc approaches to construct three-dimensional grain geometries is entirely bypassed. We name this method Granular Element Method (GEM). We demonstrate the ability of GEM in capturing, for the first time, arbitrary-shaped grains with great ease, flexibility, and without excessive geometric information. We show that the implementation of GEM is straightforward and readily integrated with standard DEM codes. Furthermore, the applicability of GEM is enhanced by its tight integration with existing computer aided geometric design and modeling tools and ability to provide a seamless transition from binary images of real grain shapes (e.g., from X-Ray CT) to modeling and discrete mechanics computations.

## INTRODUCTION

Particle morphology can be characterized, in general, by three properties: sphericity, roundness, and roughness [1]. These properties are scale dependent, as they measure morphological characteristics at different length scales, with increased spatial resolution needed to measure roughness, for example. Particle morphology has been shown to be crucially important for macroscopic properties in granular materials. In the case of macroscopic strength, it has been determined that lack of sphericity, sharper angularity and increased roughness all lead to increased mobilized strength in granular materials.

This paper presents a method within the family of the Discrete Element Method (DEM) capable of accurately capturing grain shape by using parametric curves. The new method, called the Granular Element Method (GEM) [2], accurately accounts for grain sphericity and roundness. In addition, GEM enables rapid grain shape generation and provides several significant advantages for accuracy and efficiency in computations. GEM requires minimal changes to existing DEM codes and can exploit the rapidly-improving technologies available in the computer aided geometric design industry.

### Parametric Curves for Grain Shapes

In GEM, the parametric curves have the following general form

$$\mathbf{C}(u) = \sum_{i=0}^n R_{i,p}(u) \mathbf{P}_i \quad (1)$$

where  $u$  is the curve parameter and  $R_{i,p}(u)$  are the basis functions. The form of the parametric curve is designed to provide invariance under affine transformations and strong convex hull property, as well as providing a means to calculate local curvature. These properties are important in providing flexibility in grain shape modeling and optimization using binary images of real grain shapes (e.g., from X-Ray CT), and integration with existing time integration schemes of standard DEM codes.

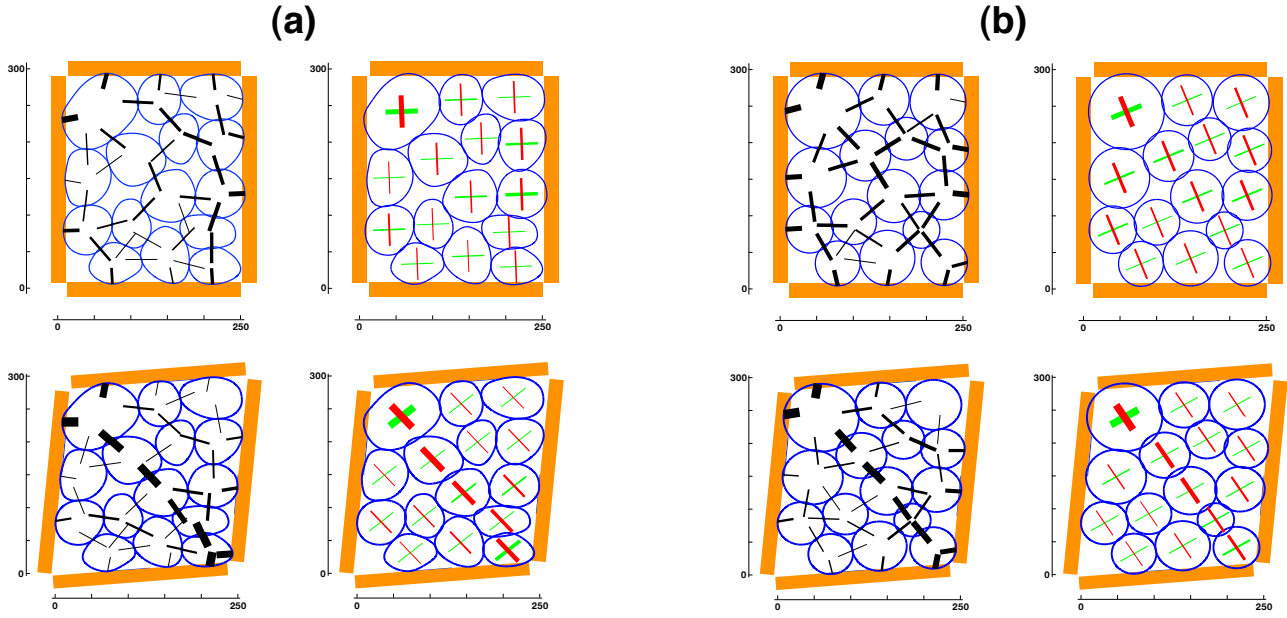
### Local Contact Resolution

For simplicity, the adopted procedure for local contact resolution and force calculation is a generalization of that utilized for circular grains [3]. For two contacting grains, two intersection points are determined. The contact plane is then calculated by connecting these two points, and the overlap is calculated from the intersection of another line which is orthogonal to the contact plane. Normal and shear forces are calculated following the same procedure as standard DEM.

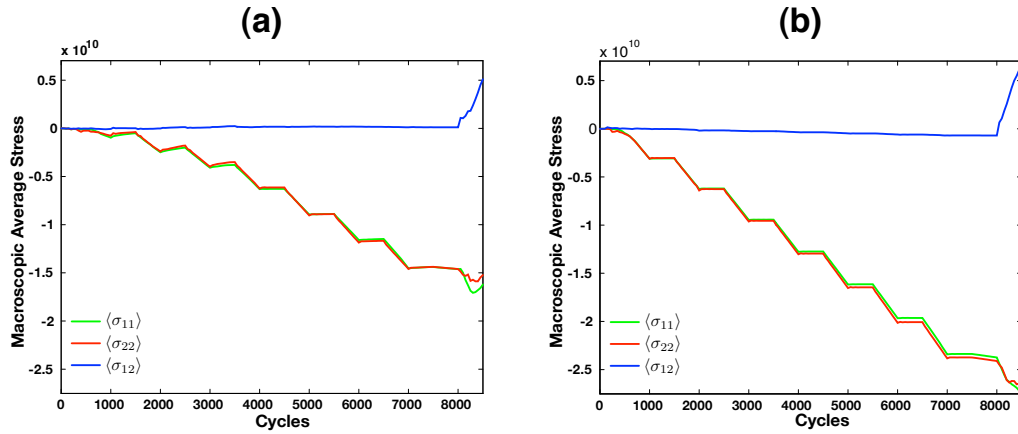
## NUMERICAL EXAMPLE

Two 15-grain assemblies, one using GEM and the other using discs, are subjected to uniform compression in several stages followed by a constant volume distortion. Figures 1 and 2 show that the macroscopic average stresses and force chains patterns are significantly different because of the different contact topologies between the two assemblies. Under uniform compression, the grains in the GEM assembly are allowed to rotate whereas the discs remained close to their initial configuration; the final macroscopic stresses in the GEM assembly are much lower than those of represented using discs. We also note that the principal directions of the average particle stresses in the two assemblies are different at the end of the uniform compression stage. The presented results showcase the ability of GEM to account for complicated load paths through a granular assembly because of the geometrical enhancement of the grain shapes provided by GEM.

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**Figure 1.** Distorted configurations for 15-grain assembly after uniform compression and distortion. (a) GEM (b) Discs. Each subfigure shows inter-particle forces (thickness proportional to force magnitude) and average principal stresses and directions in each particle (red for minor; green for major).



**Figure 2.** Evolution of average macroscopic stress components. (a) GEM (b) Discs.

## CONCLUSIONS

We have developed the Granular Element Method (GEM) for computational particle mechanics. The implementation of GEM is straightforward, with all other standard DEM procedures remaining intact. This method provides geometrical enhancement of grain shapes through the flexibility parametric curves. Grain geometrical information can be directly obtained from advanced experiments using X-Ray CT and X-Ray diffraction already available for natural granular materials such as sands. It is anticipated that GEM, coupled to experiments with X-ray CT, will be used in future experiments to infer inter-particle contact forces in real assemblies under macroscopic loads, and to connect continuum representations and the discrete states via homogenization techniques.

## References

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