A NONLINEAR HOMOGENIZATION MODEL FOR A CLAYEY ROCK HAVING A PLASTIC POROUS MATRIX

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Summary We propose a micro-macro model of plastic deformation in a clayey rock which is composed of a ductile porous clay matrix and elastic mineral grains. By making use of recent non linear homogenization methods, we derive a closed form expression of the plastic criterion which accounts for the effects of pores and of mineral inclusions. Then, a non associated flow rule is proposed. Comparisons between the predictions of the full micro-macro model and available experimental data show that it is able to capture the main features of the mechanical behavior of the clayey rock.

ON THE MODELING OF THE CALLOVO-OXFORDIAN (COX) ARGILLITE

The Callovo-Oxfordian (COx) argillite is a clayey rock involved in the French national research program dealing with the feasibility study of underground waste storage. This rock is characterized by its low hydraulic conductivity and relatively high mechanical strength. Its mineralogical composition varies with the in situ depth and contain three main phases: clay matrix (40 to 50%), calcite (20 à 27%) and quartz grains (23 à 25%). The porosity of COx argillite varies from 11 to 14%. For simplicity, the argillite is schematized at mesoscale as a two phase composite: effective inclusions embedded in the clay matrix. The effective inclusions are assumed elastic rigid, spherical and randomly distributed. The clay matrix is an isotropic elastoplastic porous medium with a typical local porosity value $f = 25\%$.

MACROSCOPIC CRITERION OF POROUS MEDIA REINFORCED BY RIGID INCLUSIONS

According to the above description, the studied material at mesoscale is composed of two phases: the rigid elastic inclusions and the embedding elastoplastic porous matrix. In the Representative elementary volume Ω, the volume fraction of pores, rigid inclusions and of the solid matrix are respectively denoted: Ω_p, Ω_i and Ω_m. The porosity f of the matrix and the absolute volume fraction ρ of the rigid inclusions reads: $f = \frac{\Omega_p}{\Omega_p+\Omega_m}$, $\rho = \frac{\Omega_i}{\Omega_i+\Omega_m+\Omega_p}$.
The solid phase of the matrix obeys to a Drucker-Prager criterion:

$$\phi^m(\tilde{\boldsymbol{\sigma}}) = \tilde{\sigma}_d + T(\tilde{\sigma}_m - h) \leq 0 \tag{1}$$

$\tilde{\boldsymbol{\sigma}}$ denotes the local stress in the solid phase, $\tilde{\sigma}_m = \mathrm{tr}\tilde{\boldsymbol{\sigma}}/3$ being mean stress, $\tilde{\sigma}_d = \sqrt{\tilde{\boldsymbol{\sigma}}' : \tilde{\boldsymbol{\sigma}}'}$ generalized deviatoric stress with $\tilde{\boldsymbol{\sigma}}' = \tilde{\boldsymbol{\sigma}} - \tilde{\sigma}_m\mathbf{1}$. The symbol "$\sim$" is used in order to make difference with the microscopic stresses of the solid phase and the mesoscopic stress field attached to the porous medium. The parameter h represents the hydrostatic tensile strength while T denotes the frictional coefficient.
By using a non linear homogenization technique based on the modified secant method, [1] has shown that, even in the context of non associated solid phase, the yield criterion of the porous medium having a Drucker-Prager matrix reads as in the associated case:

$$F^{mp}(\boldsymbol{\sigma}, f, T) = \frac{1 + 2f/3}{T^2}\sigma_d^2 + \left(\frac{3f}{2T^2} - 1\right)\sigma_m^2 + 2(1-f)h\sigma_m - (1-f)^2h^2 = 0 \tag{2}$$

In the present study, this criterion will be used to describe the yield of the elastoplastic behavior of the porous matrix. It remains to build the macroscopic criterion of the whole composite. Since the strength domain $F^{mp}(\boldsymbol{\sigma}, f, T)$ is convex and closed in the stress space, the support function π^{mp} in the porous medium can be expressed as function of the volumetric deformation d_v and the norm of the deviatoric deformation d_d. In the perspective of application of the modified secant moduli approach, the stress-strain relationship derived from this support function π^{mp} can be put in the following form:

$$\boldsymbol{\sigma} = \frac{\partial \pi^{mp}}{\partial \boldsymbol{d}} = 2\mu^{mp}\boldsymbol{d}' + \kappa^{mp}d_v\mathbf{1} + \sigma^p\mathbf{1} \tag{3}$$

κ^{mp}, μ^{mp} being the secant bulk and shear moduli, respectively and σ^p denoting a spherical prestress. The secant moduli are non-uniform, due to their dependence on the heterogeneity of the strain rate field $\boldsymbol{d}$. Following [1], we consider the effective strain rate $\boldsymbol{d}^{eff}$. and the effective moduli $\kappa^{mp} = \kappa^{mp}_{eq}(d_v^{eff}, d_d^{eff})$, $\mu^{mp} = \mu^{mp}_{eq}(d_v^{eff}, d_d^{eff})$ and the spherical prestress $\sigma^p = \sigma^p_{eq}(d_v^{eff}, d_d^{eff})$. These quantities depend on the macroscopic strain rate $\boldsymbol{D}$.
Due to the matrix-inclusions morphology of the material, the macroscopic state equation of the composite can be obtained by using the Hashin-Shtrikman lower bound in the case of rigid inclusions. It follows that:

$$\kappa^{hom} = \frac{3\kappa^{mp}_{eq} + 4\rho\mu^{mp}_{eq}}{3(1-\rho)}, \quad \mu^{hom} = \mu^{mp}_{eq}\frac{\kappa^{mp}_{eq}(6+9\rho) + \mu^{mp}_{eq}(12+8\rho)}{6(1-\rho)(\kappa^{mp}_{eq}+2)\mu^{mp}_{eq}} \tag{4}$$

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The effective volumetric and deviatoric strain rates in the porous matrix are defined by the modified secant method as: $d_v^{eff} = \sqrt{< d_v^2 >_{\Omega^{mp}}}$, $d_d^{eff} = \sqrt{< d_d^2 >_{\Omega^{mp}}}$. Following a methodology used by [2], the effective strain rates in the equivalent porous matrix are obtained as:

$$\frac{1}{2}(1-\rho){d_v^{eff}}^2 = \frac{1}{2}\frac{\partial \kappa^{hom}}{\partial \kappa_{eq}^{mp}} D_v^2 + \frac{\partial \mu^{hom}}{\partial \kappa_{eq}^{mp}} D_d^2, \quad (1-\rho){d_d^{eff}}^2 = \frac{1}{2}\frac{\partial \kappa^{hom}}{\partial \mu_{eq}^{mp}} D_v^2 + \frac{\partial \mu^{hom}}{\partial \mu_{eq}^{mp}} D_d^2 \tag{5}$$

The resulting macroscopic criterion (of the material made up of the porous matrix and rigid inclusions) takes the form:

$$\Phi(\mathbf{\Sigma}, f, T) = \frac{\frac{1+2f/3}{T^2} + \frac{2}{3}\rho\left(\frac{3f}{2T^2} - 1\right)}{\frac{4T^2-12f-9}{6T^2-13f-6}\rho + 1}\Sigma_d^2 + \left(\frac{3f}{2T^2} - 1\right)\Sigma_m^2 + 2(1-f)h\Sigma_m - \frac{3+2f+3f\rho}{3+2f}(1-f)^2h^2 = 0 \tag{6}$$

HOMOGENIZATION-BASED NON ASSOCIATED FLOW RULE FOR THE ARGILLITE

In order to complete the formulation of the micro-macro model, we aim at proposing a non associated flow rule for the COx clayey rock; inspired by the mathematical form of (6), the following plastic potential is suggested:

$$G(\mathbf{\Sigma}, f, \bar{T}, \bar{t}) = \frac{\frac{1+2f/3}{\bar{T}\bar{t}} + \frac{2}{3}\rho\left(\frac{3f}{2\bar{T}\bar{t}} - 1\right)}{\frac{4\bar{T}\bar{t}-12f-9}{6\bar{T}\bar{t}-13f-6}\rho + 1}\Sigma_d^2 + \left(\frac{3f}{2\bar{T}\bar{t}} - 1\right)\Sigma_m^2 + 2(1-f)h\Sigma_m - \frac{3+2f+3f\rho}{3+2f}(1-f)^2h^2 = 0 \tag{7}$$

in which t represents the dilatancy coefficient.
Concerning the plastic hardening of the solid phase, modeled via the evolution of the frictional coefficient T and that of the dilatancy coefficient t with the equivalent plastic strain ε^p, the following forms are proposed:

$$\bar{T} = T_m - (T_m - T_0)e^{-b_1\varepsilon^p}, \bar{t} = t_m - (t_m - t_0)e^{-b_2\varepsilon^p} \tag{8}$$

VALIDATION OF THE MICRO-MACRO MODEL FOR THE ARGILLITE

A series of uniaxial and triaxial compression tests are now simulated using the proposed model with the non associated flow rule. Different confining pressures and mineralogical compositions are considered. The model predictions are compared with available experimental data as shown in Figures 1 and 2. A good agreement is observed. Various other tests are simulated and shown in [3]

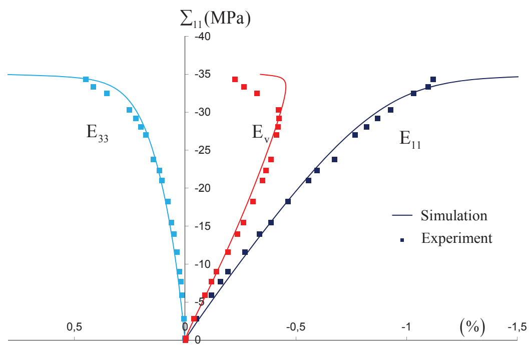

Figure 1. Depth: 466.8m, $f_0 = 51\%$, $f_1 = 49\%$, Uniaxial compression test

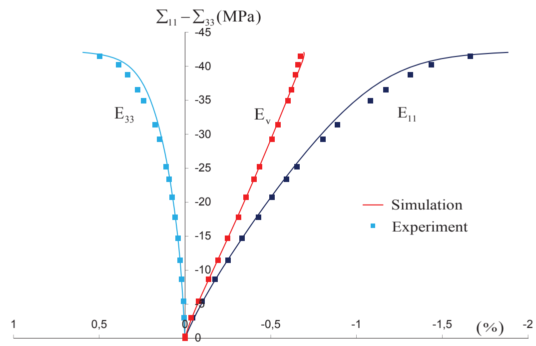

Figure 2. Depth: 482.2m, $f_0 = 60\%$, $f_1 = 40\%$, triaxial compression test with confining pressure of $10MPa$.

CONCLUSIONS

We have proposed an homogenization-based model for the elastoplastic behavior of the COx argillite. At the mesoscopic scale, this material is considered as a two phase composite: porous clay matrix embedding mineral inclusions. Two homogenization steps are carried out and allow to formulate at macroscopic scale a closed form expression of the non associated constitutive law which accounts for the effects of mineral inclusions and the influence of porosity. The predictions of the model agree with the available experimental data.

References

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