A 2D CONSTITUTIVE MODEL WITH ANISOTROPY FOR GRANULAR MATERIALS

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Summary The non-coaxiality of stress, strain and structure is an essential part of a constitutive model for granular matter because it describes how the deformation paths have affected the mechanical state of the system. The paper presents a model for two-dimensional anisotropic granular materials, based on non-linear tensorial functions. The history dependence is represented by a symmetric 2nd-rank fabric tensor, which characterizes the geometric arrangement of the grains.

Introduction

To understand the behavior of granular materials is first necessary to characterize their state by introducing appropriate internal variables. In early studies, macroscopic variables measurable in laboratory experiments, i.e. stress and porosity, were thought to be sufficient. However with further investigations it has been determined that other factors, such as the average orientational distributions of the grains, i.e. the geometric fabric [6], are also important. It determines the anisotropic behavior of the material and endows it with a memory of its previous states. Anisotropy in granular materials may exist before straining (inherent or depositional fabric) or it may be induced by strain. In the first case, samples initially of the same density behave differently on loading, depending on the orientation of the shearing direction relative to the direction of the deposition. In the second case, identical samples brought to the same stress and density in different ways will show subsequent differences in behavior [8]. This experimental evidence show that fabric is not totally controlled by the stress [2, 8].

Numerical observations

We carefully investigate the relation between macroscopic observable (moduli) anisotropy and fabric anisotropy, described through a 2nd-rank tensor [6]. To do this, we use DEM [1] simulations to create numerical samples characterized by stress isotropy and anisotropic distribution of contacts (transversally isotropic with axes $\mathbf{e}_3$). Each of the four samples $S_1 - S4$ is then isotropically compressed leading to sixteen equilibrated states after different isotropic strain. We measure the deviatoric fabric in the samples and the macroscopic anisotropy given by the difference between the shear moduli $G_{13} \simeq G_{23}$ and G_{12}. Results are plotted in Fig. 1. Since the material is isotropic in stress, the different shear response in the axial and transversal directions can be only due to the fabric.

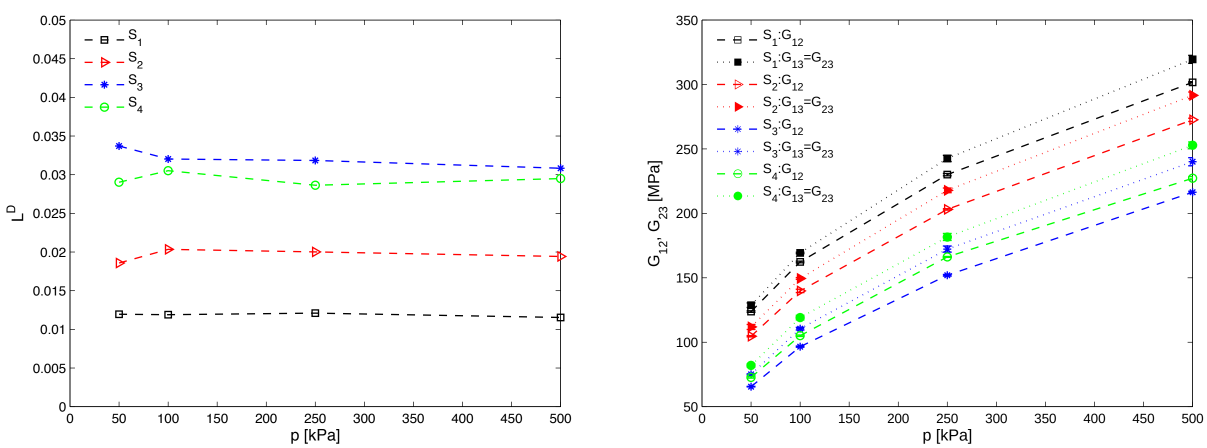

Figure 1. Deviatoric fabric L^D and shear moduli versus confining pressure.

Model

The inclusion of the concept of a dynamic (history-dependent parameter) material structure tensor into continuum models [7], or its relation to advanced hypoplastic continuum constitutive models [4] with internal tensorial state variables [3], is still an open issue. As further step, we present a constitutive model for two-dimensional stress-strain behavior of anisotropic granular materials, based on non-linear tensorial evolution equations. We distinguish the effects of stress and fabric anisotropy and use them in order to properly model the constitutive behavior of a granular material.

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We describe the evolution of stress through the following constitutive equation of rate-type

$$\overset{\circ}{\mathbf{T}} = \mathbf{H}(\mathbf{T}, \mathbf{L}, \mathbf{D}), \tag{1}$$

where we express the corotational stress rate $\overset{\circ}{\mathbf{T}}$ as function of the stress state $\mathbf{T}$, the fabric tensor $\mathbf{L}$, and the strain rate $\mathbf{D}$. We request the 2D function to be objective [5] and positively homogeneous of first degree in $\mathbf{D}$. Together with this, we model the evolution of the fabric as

$$\overset{\circ}{\mathbf{L}} = \mathbf{K}(\mathbf{L}, \mathbf{D}). \tag{2}$$

$\overset{\circ}{\mathbf{L}}$ obeys to similar restrictions as $\overset{\circ}{\mathbf{T}}$ but it is only a function of geometric, kinematic quantities. Stress and fabric evolve independently. Coupling Eqs. (1) and (2), with proper calibration of the material parameters through DEM simulations, we are able to model the full stress-strain behavior in a 2D granular assembly.

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