MICROMECHANICAL FORMULATION OF FORCE TRANSPORT IN WET GRANULAR MEDIA AND STRENGTH ISSUES

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Summary The mechanical behaviour of a wet granular material is investigated through a micromechanical analysis of force transport between interacting particles with a given packing and distribution of capillary liquid bridges. We derive a single effective stress tensor encapsulating evolving liquid bridges, packing, interfaces, and water saturation providing a proper link to failure. A deviatoric suction stress arises from the formulation and its implications on strength and failure are investigated.

INTRODUCTION

Wet granular materials at the low end of degree of saturation (less than 30%), i.e. in the pendular regime, represent a three-phase system whereby the liquid phase is discontinuous in the form of isolated liquid bridges (menisci). As such, rich material behaviours at the macroscopic scale arise due to the interplay between capillary forces within the water meniscus and interparticle contact forces. This is made evident by the case where, during an increase in moisture and temperature, contact forces are disrupted in combination with a loss in capillary forces to give way to collapse failure in the absence of any change in mechanical loading. In determining the behaviour and strength characteristics of such unsaturated granular media, it becomes difficult to choose the controlling stress variable that would substitute for the role of the effective stress in fully saturated media. The stress-strain behaviour is in fact closely related to moisture content and more precisely, the distribution of liquid menisci and particle packing. Bishop [1] intuitively extended Terzaghi's effective stress principle to account for the presence of an air phase by introducing an average pore fluid pressure weighted over the pore air (u_a) and water (u_w) pressures, i.e.

$$\sigma' = \sigma - \left[\chi u_w + (1-\chi)u_a\right] = (\sigma - u_a) + \chi(u_a - u_w) \quad \text{(positive stresses mean compression)} \tag{1}$$

where σ and σ' are the total and effective stresses respectively, and χ is a weighted parameter that is arbitrarily confounded with the degree of water saturation, S_r. Understanding the suction stress as a function of the difference between air and water pressures, menisci distribution, as well as its dependency on the degree of saturation and particle packing is a longstanding problem both theoretically and experimentally. This issue establishes a direct connection between constitutive behaviour and strength of unsaturated media in providing an adequate expression for effective stresses.

STRESS TRANSPORT IN THREE-PHASE MEDIUM

We consider a three-phase system composed of an ensemble of mono-disperse spherical particles of radius R joined by independent concave liquid bridges in a representative elementary volume (REV) V in equilibrium with applied loads on its boundaries. By considering air and water pressures, surface tension, as well as interparticle forces within the REV, the (Cauchy) stress tensor can be readily calculated as a volume average of the various constituents (phases) just as in the case of a solid body consisting of interacting point masses in a representative elementary volume [2]. Without elaborating on details of the derivation, we eventually get an effective stress tensorial equation such as:

$$\sigma'_{ij} = (\sigma_{ij} - u_a\delta_{ij}) + \chi_{ij}(u_a - u_w);\ \chi_{ij} = \theta\delta_{ij} + \frac{R}{V}\sum^{N}\sum^{L}\int_{\Gamma_w^p} n_j n_i d\Gamma \tag{2}$$

in which θ is the volumetric water content, N the number of particles, L the number of liquid bridges, whereas χ_{ij} is a tensored value material quantity which is anisotropic and serves as an analogue to χ in the original Bishop's equation (1). The second-order tensor χ_{ij} is interpreted as a distributional description of liquid bridges through a surface integral of dyadic products of contact normals n_i and n_j calculated over Γ_w^p, the wetted surface area of a specific particle. Within this micromechanical framework, it is evident that we recover Bishop's equation with two independent state variables: $(\sigma_{ij} - u_a\delta_{ij})$ and suction stress $\chi_{ij}(u_a - u_w)$, but with the difference that the latter can be anisotropic in nature, thus in opposition to common opinion.

SUCTION STRESS AND STRENGTH FOR SIMPLE PERIODIC PACKINGS

The effective stress parameter χ for isotropic liquid bridge distribution as a function of degree of water saturation S_r, is theoretically evaluated for two limiting cases, namely simple cubic (SCP) packing (loosest state) and face centered (FCP) packing (densest state) in idealized soil with mono-sized spherical particles, see Fig. 1a together with actual experimental data comparisons. Figs. 1b,c show SCP and FCP geometries respectively with liquid bridges between each pair of particles.

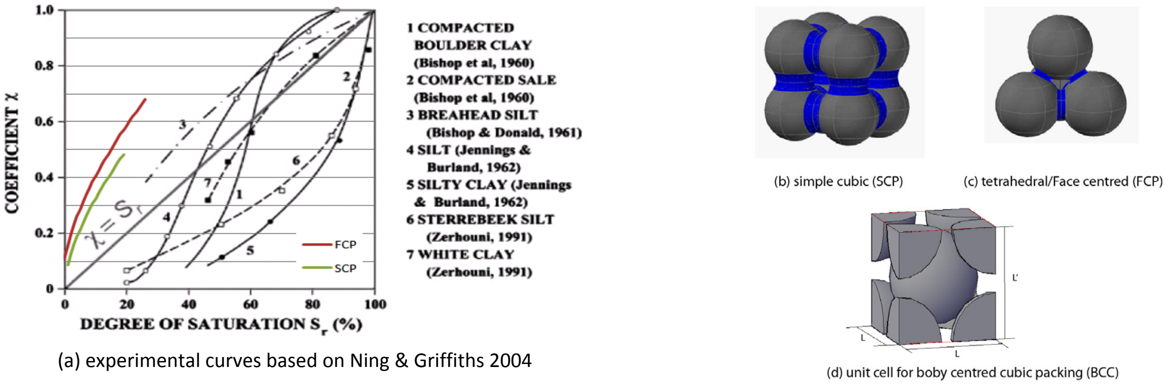

Figure 1. Computed relationships between degree of saturation Sr and effective stress parameter χ for various packings

A body centred cubic packing (BCC) is next evoked with its unit cell at a coordination number of 8 as shown in Fig. 1d. By varying the size of the unit cell through lengths L and L', a family of anisotropic BCC packings with different particle separations can be generated. The ensuing anisotropic suction stress engenders a meniscus based shear strength (friction angle) in triaxial extension which can be computed from Eq. (2) with Mohr-Coulomb. Fig. 2 shows that the friction angle (φ_s) mobilized by suction increases with degree of anisotropy. As expected, φ_s decreases with saturation and becomes zero in the isotropic case where the shear contribution of the menisci disappears. Depending on the packing anisotropy, this friction angle drop with water saturation can be drastic so as to cause a collapse failure.

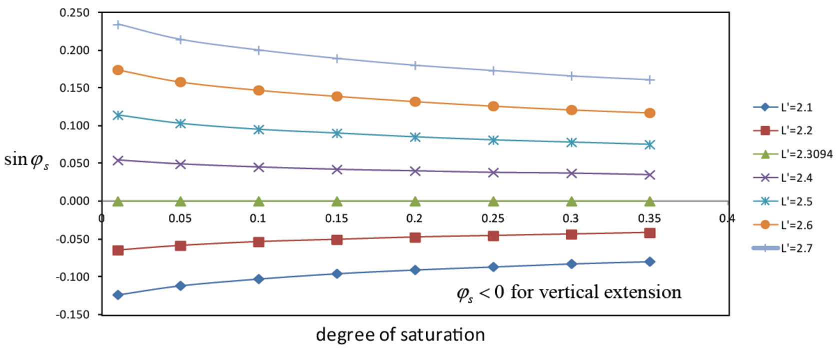

Figure 2. Friction angle induced by water menisci as a function of saturation and anisotropy of BCC packing

As a further support of the validity of the derived generalized effective stress equation, the computation of effective stress based on Eq. (2) for various water contents considering menisci and particle packing effects leads to a unique Mohr-Coulomb failure envelope with an intrinsic friction angle as for the dry case (see, Fig. 3). The experimental data used in the computations are based on a mono-dispersed assembly of spheres in the pendular regime for which the shear strength is known at various water contents, see [3].

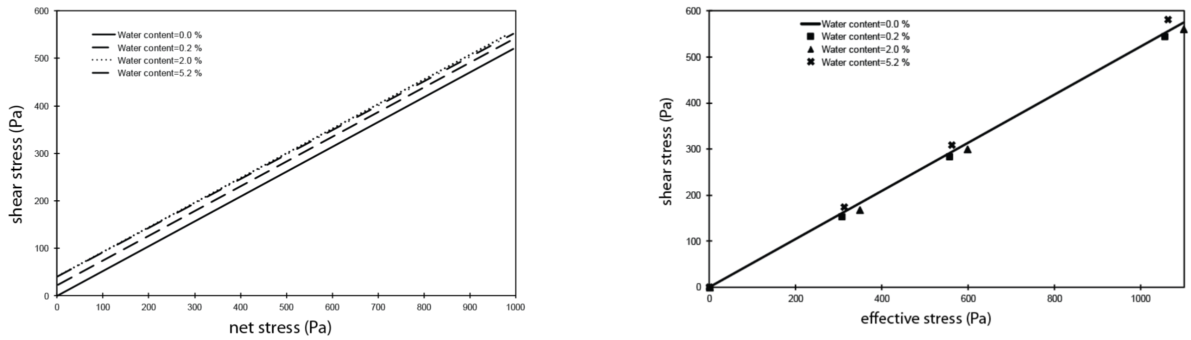

Figure 3. Strength of wet granular assembly based on: (a) net stress, (b) effective stress following Eq. (2).

References

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