TOWARDS RELIABLE AND EFFICIENT ALGEBRAIC COARSE-GRAINING OF PARAMETRIC PROBLEMS IN FRACTURE MECHANICS

Pierre Kerfriden* & Stéphane Bordas*
*Cardiff University, School of Engineering, Queen's Buildings, The Parade, Cardiff CF24 3AA, Wales, UK

Summary We present in this paper a novel methodology to coarse-grain fine-scale problems in fracture mechanics. In order to enable the prediction of initiation and propagation of macroscopic defects, the material behaviour of the considered structure is described at a fine-scale (e.g.: the scale of the material heterogeneities). Such a description leads to untractable numerical problems at an engineering scale. We investigate the use of algebraic coarse-graining of the fine-scale problem via the Proper Orthogonal Decomposition. We show that problems in fracture mechanics exhibit a lack of correlation, which prevents the straight use of reduced order modelling techniques. However, the lack of correlation can be spatially isolated and efficient algebraic coarse-graining schemes can be derived from this observation. Specifically, we couple reduced order modelling and domain decomposition, and demonstrate that different reduction schemes can be used adaptively in different subdomains, depending on the corresponding level of correlation.

INTRODUCTION

Engineering structural problems are often characterised by a large ratio between the scale of the structure and the scale at which the phenomena of interest are described. In fracture mechanics, the material failure is described at the scale of the structural heterogeneities. Such a mismatch results in large numerical models, a discretisation at the fine scale over the whole structure being necessary to represent local gradients. The solution to such problems using super-computers is now at reach, but at a considerable price. However, in engineering design processes, a prohibitively high number of such solutions might be of interest, for different values of design parameters, or to take into account the effect of stochastic phenomena. Consequently, coarse-graining multiscale engineering problem is an issue of tremendous importance in today's computational mechanics.

Model order reduction techniques that are based on the projection of fine scale problems in reduced spaces are a potential solution to this issue. Such strategies assume that the fine-scale solutions obtained for different values of the input parameters are accurately approximated in low-dimensional subspaces spanned by well-chosen global basis functions. Applying this idea, the numerous unknowns that arise from the discretisation of the fine-scale problem are reduced to a few global state variables. Such strategies have been extensively studied in various field of engineering, from the linear case to "mildly" nonlinear cases [1, 2]. Some attempts have been proposed to extend this concept to strong nonlinearities, in particular in structural mechanics [3, 4, 5, 6]. This background makes it conceivable to use such methods in complex, highly nonlinear engineering problems such as fracture mechanics.

Fracture mechanics is characterised by an intrinsic lack of separation of scales between the engineering scale and the scale at which damage initiation is described. Consequently, these problems are not directly reducible by the aforementioned techniques. More precisely, the level of reducibility of such multiscale problems depends on the region of the domain which is considered. Typically, the solution in the zones were damage initiates and propagates will not be correctly approximated in low-dimensional subspaces (this fact will be illustrated in the core of the paper). To circumvent this difficulty, the idea presented in this paper is to use a partitioning of the domain into subdomains in order to perform a systematic model order reduction of the resulting subproblems (similar to the idea underlying the classical Craig-Bampton method, though in a different context). In the case where subproblems are not reducible due to strong local nonlinearities and lack of scale separation, indicators will permit to bypass model reduction for the corresponding subdomain.

METHODOLOGY

More specifically, we propose to obtain a coarse-grain model by the snapshot-proper orthogonal decomposition (POD), combined with a Schur-based domain decomposition solver (see a review in[6], and an adaptive usage, relevant to the present work, to solve damage problems in vii). During an "offline" stage, a set of fine-scale solutions corresponding to well-chosen valued of the input parameters, called snapshot, is generated in parallel using the domain decomposition technique. POD transforms are performed locally to obtain attractive low-dimensional approximation spaces for the solution to the subproblems. In the "online" stage, the solution corresponding to further values of the input parameter is looked for at reduced costs in the generated local reduced spaces. The coupling between the reduced subproblems is obtained by the same domain decomposition method used to generate the snapshot, in a relatively transparent manner.

The reduced spaces are constructed adaptively, depending on the nature of the subproblems. Subdomains undergoing crack propagation are algebraically identified by a poor convergence of the POD transform as a function of (i) the order of the expansion and (ii) the dimension of the snapshot. In this case, a criterion is proposed to switch to a direct local solver. For subdomains exhibiting a satisfying convergence of the POD, a suitable truncation of the POD basis is defined, along with a stagnation criterion for the convergence of the corresponding attractive subspace of the snapshot. Subdomains undergoing higher level of nonlinearities are automatically granted richer reduced spaces, both in terms of order of the

a) Corresponding author. E-mail: ictam2012@cstam.org.cn.

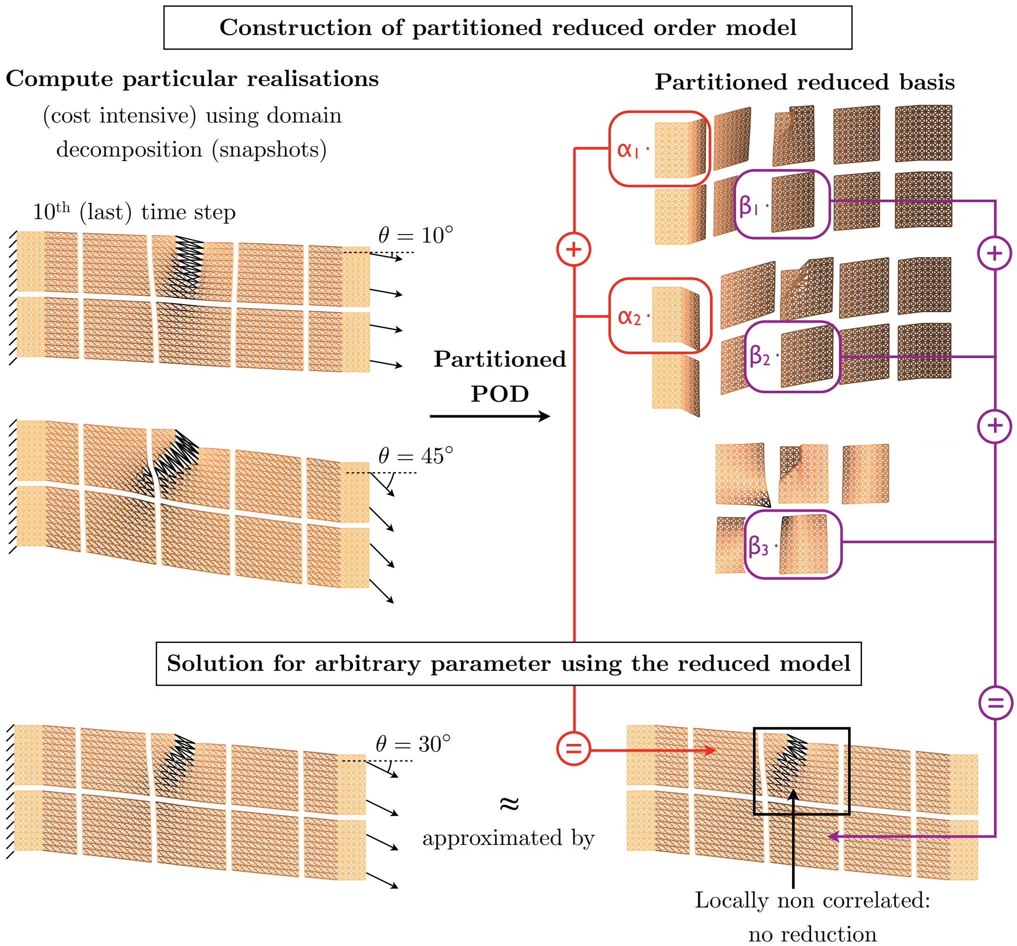

Figure 1. Illustration of the proposed partitioned approach to POD-based reduced order modelling.

expansion, and values of the input parameters used to generate the snapshot.

RESULTS AND CONCLUSIONS

The proposed adaptive scheme permits to localise the numerical effort where it is most needed, in the process zones. Elsewhere, algebraically coarse-grained problems are solved at reduced costs. As a result, a significant speed-up is obtain for parametric problems of fracture. The speed-up is highly dependant on the ratio between the size of the process zone, for which an approximation space cannot be reliably used, and the size of the structure.

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