THE RAPIDEST UNLOADING IN BRITTLE FRAGMENTATION PROCESS

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Summary A theoretical model was established to study the unloading of a brittle solid undergoing fragmentation, and to evaluate the average fragment size. Assuming that the fragmentation of a solid is the simultaneous opening of an array of equally-spaced cracks, the unloading process of the solid due to crack developments is investigated. The critical time when the average stress drops to zero is determined. For any prescribed strain rate, there exists an optimum crack spacing that corresponds to the rapidest unloading of the solid. Assuming that in a natural fragmentation process the brittle solid is unloaded in the fastest way, the average fragment size can be estimated through this optimum crack spacing, which agrees fairly well with the previous results of random crack simulations.

Keywords brittle fragmentation; crack array; the method of characteristics; the rapidest unloading; fragment size

INTRODUCTION

Brittle solids usually break into many pieces (fragmentizes) during a high strain rate expansion. Many investigations have conducted to predict the average size of the fragments [1~8]. Although the multi-fracture (fragmentation) of brittle solids is intrinsically stochastic, there exist some relationship between the average fragment size and the material/loading parameters. Through numerical simulations of random (*natural*) fragmentations, we have conducted detailed investigations of the influence of material parameters on the rate-dependence of fragment size, and obtained a general scaling law to calculate the fragment sizes for brittle materials [9]. In this paper, we aim to study the unloading of the expanding solids during the fragmentation process, and to develop a new method to evaluate the average fragment size.

MODEL AND GOVERNING EQUATION

Under the one-dimensional uniform tensile strainrate $\dot{\varepsilon}$, a brittle solid breaks into many pieces in the loading direction. We treat the fragmentation of the solid as the simultaneous opening of an array of cracks equally-spaced with distance *L* (Fig 1a). By symmetry, a unit body containing one crack is analyzed (Fig 1b). At the cracking point, a linear cohesive fracture law is used to describe the opening behavior of the cracks (Fig 1c). The interactions between the cracks, and the unloading of the solid are controlled by elastodynamics of the unbroken elastic media. In the region $0 \le x \le L/2$, the stress $\sigma(x,t)$ and velocity $v(x,t)$ satisfy the following PDEs and the initial/boundary conditions:

$$\begin{aligned}&\rho\,\partial v/\partial t = \partial\sigma/\partial x\ ,\ \ \partial\sigma/\partial t = E\,\partial v/\partial x \qquad (0\le x\le L/2)\\ &\sigma(x,0)=\sigma_c\ ,\ v(x,0)=\dot{\varepsilon}x\ \ (0\le x\le L/2)\ ;\ \ \dot{\sigma}(0,t)/\sigma_c=-\sigma_c v(0,t)/G_c\ ,\ v(L/2,t)=\dot{\varepsilon}L/2\quad (0\le t\le t_f)\end{aligned} \tag{1}$$

where $E=\rho c^2$ is Young's modulus, *c* the elastic wave speed, σ_c the failure strength and G_c the fracture energy.

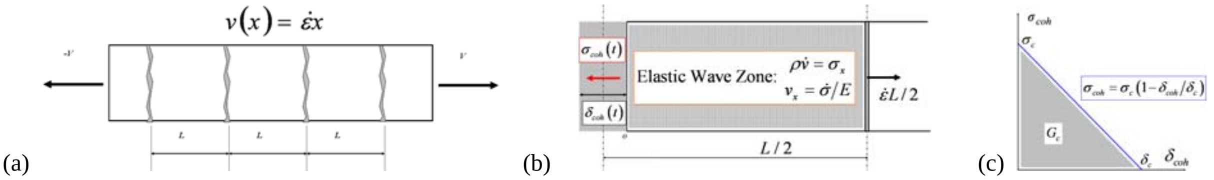

Fig. 1(a) An array of equally spaced cohesive cracks; (b) the region of analysis; (c) the linear cohesive fracture model

SOLUTION STRATEGY

Within the limit of linearity, (1) can be solved by applying Laplace transform to the governing PDEs, solving the resulted ODEs, and inverse transforming the solutions to the time domain. Detailed information on the solution procedures is omitted. As the result of the solution, the stress at the cracking point is:

$$\frac{\bar{\sigma}_{coh}(\bar{t})}{\bar{\sigma}_c}=1+\frac{\bar{\dot{\varepsilon}}}{\bar{\sigma}_c}\bar{t}+\sum_{n=0}^{\infty}q_n(\bar{t})\quad ,\quad q_n(\bar{t})=2\left(\bar{\dot{\varepsilon}}/\bar{\sigma}_c^{\,2}\right)\mathbf{L}^{-1}\left[\frac{(1/\bar{\sigma}_c+s)^{n-1}}{s(1/\bar{\sigma}_c-s)^{n+1}},s\to\bar{t}\,'=(\bar{t}-n\bar{L})\right]\mathbf{H}(\bar{t}-n\bar{L}) \tag{2}$$

where the nondimensional quantities are defined as: $\bar{\sigma}=\sigma/E$, $\bar{\sigma}_c=\sigma_c/E$, $\bar{t}=t/\left(E^2G_c/c\sigma_c^{\,3}\right)$, $\bar{\dot{\varepsilon}}=\dot{\varepsilon}/\left(c\sigma_c^{\,3}/E^2G_c\right)$, and $\bar{L}=L/\left(E^2G_c/\sigma_c^{\,3}\right)$. $\mathbf{H}(\ )$ is the Heaviside function, $\mathbf{L}^{-1}\left[f(s),s\to\bar{t}\right]$ the inverse Laplace transform. In (2) each term $q_n(\bar{t})=q_n{}'(\bar{t}-n\bar{L})\mathbf{H}(\bar{t}-n\bar{L})$ represents superimposed stresses due to repeated elastic wave reflections between the neighboring cracks. The kernel part of the inverse transform $\mathbf{L}^{-1}\left[(1/\bar{\sigma}_c+s)^{n-1}/s(1/\bar{\sigma}_c-s)^{n+1},s\to\bar{t}\,'\right]$ can be readily obtained by using the residue theorem for the Bromwich integration. The average stress history is given by:

$$\frac{\bar{\sigma}_{ave}(\bar{t})}{\bar{\sigma}_c}=1+\frac{\bar{\dot{\varepsilon}}}{\bar{\sigma}_c}\bar{t}+\sum_{n=0}^{\infty}a_n(\bar{t})\ ,\ a_n(\bar{t})=\frac{4\left(\bar{\dot{\varepsilon}}/\bar{\sigma}_c\right)}{\bar{L}}\mathbf{L}^{-1}\left[\frac{(1/\bar{\sigma}_c+s)^{n-1}}{s(1/\bar{\sigma}_c-s)^{n+1}},s\to\bar{t}\,'=(\bar{t}-n\bar{L})\right]\mathbf{H}(\bar{t}-n\bar{L}) \tag{3}$$

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The exact solutions (2, 3) apply for the time before the cracking point is completely broken ($\bar{t} \le \bar{t}_f$), $\bar{t}_f$ can be determined by setting $\bar{\sigma}_{coh}(\bar{t})$ in (2) to zero. The expression in (3) gives the history of the average stress in the solid up to $\bar{t} \le \bar{t}_f$. Due to the complex elastic wave interactions, the stress within the solid at the time $\bar{t}_f$ is not necessarily zero. The time when the average stress decreases to zero, $\bar{t}_c$ is viewed as the event the solid is completely unloaded. This time can be determined by numerical analysis based on the method of characteristics.

RESULTS AND CONCLUSIONS

Fig. 2 plots the average stress histories, at the prescribed nondimensional strainrate $\dot{\bar{\varepsilon}}$ =0.01, 0.1, 1.0, and 10. The time $\bar{t}_c$ is different for different crack spacing $\bar{L}$. For each prescribed strainrate $\dot{\bar{\varepsilon}}$, the $\bar{t}_c \sim \bar{L}$ curve is plotted in Fig. 3a. All these curves are U-shaped curve. At very small $\bar{L}$ the time $\bar{t}_c$ is larger, as the very closely nucleated cracks interfere with each other so that they are not easy to grow. At very large $\bar{L}$, $\bar{t}_c$ becomes larger again, as the complete unloading of the solid takes more time for the unloading wave to propagate through the unbroken part. For each U-shaped $\bar{t}_c \sim \bar{L}$ curve, there is a bottom point representing the least $(\bar{t}_c)_{Min}$, or the rapidest unloading of the solid. The crack spacing at this bottom point $(\bar{L})_{Opt}$, is the optimum spacing of the rapidest unloading process.

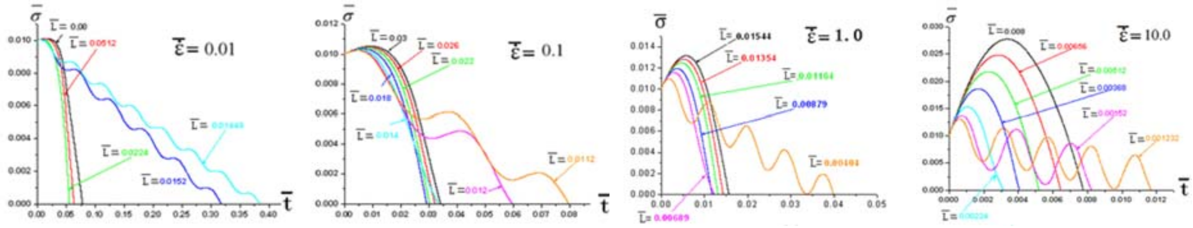
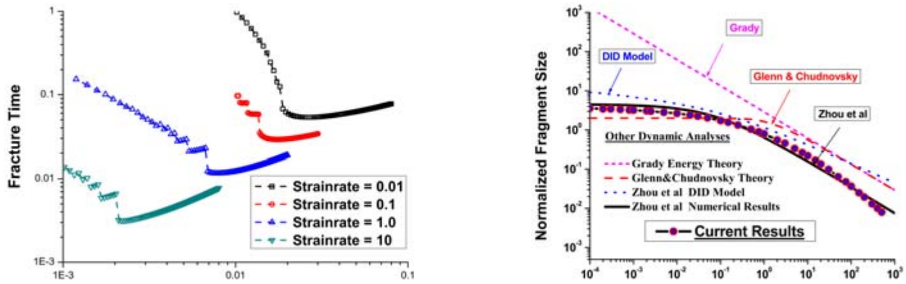


Fig. 2 History curves of the average stress at prescribed strainrates: $\dot{\bar{\varepsilon}} = 0.01, 0.1, 1, \text{and } 10$



(a) (b)

Fig. 3 (a) The U-shaped $\bar{t}_c \sim \bar{L}$ curves under $\dot{\bar{\varepsilon}} = 0.01, 0.1, 1, \text{and } 10$; (b) The $\dot{\bar{\varepsilon}} \sim \bar{s}$ curves

Physically, the fragmentation of a brittle solid is a natural process. Assume that as a solid is tensioned at constant $\dot{\bar{\varepsilon}}$, the nucleated cracks will automatically adjust their spacings so that the solid is unloaded within the least time. If this assumption is true, the optimum crack spacing at each curve in Fig. 3a is the average fragment size at the prescribed strainrate. To verify this conjecture, we investigated the multiple crack fracture process at $\dot{\bar{\varepsilon}} = 10^{-4} \sim 5 \times 10^{2}$, at each strainrate the average stress unloading curve is evaluated for a range of crack spacings, obtaining the optimum crack spacing $(\bar{L})_{Opt}$. We plot in Fig. 3b the obtained $(\bar{L})_{Opt}$ against the applied $\dot{\bar{\varepsilon}}$. These points, surprisingly, fall very close to the nondimensional “fragment size – strainrate” curve, which has been obtained by Zhou et al. through systematic numerical simulations [9]. It is shown that the natural brittle fragmentation processes occur so that the deforming solid is unloaded the rapidest way. The mechanism of the “rapidest unloading” may be a fundamental feature of dynamic fragmentation, and this mechanism can be used as an alternate way to estimate the average fragment size in other loading and deformation configurations.

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References

[1] Mott N.F. (1947), *Proceedings of Royal Society London*, **A-189**, 300-308
[2] Grady D.E. (1982), *Journal of Applied Physics*, **53**, 322-325
[3] Glenn L.A. and Chudnovsky A. (1986), *Journal of Applied Physics*, **59**, 1379-1380
[4] Miller O., Freund L.B. and Needleman A. (1999), *International Journal of Fracture*, **96**, 101-125
[5] Drugan W.J. (2001), *Journal of the Mechanics and Physics of Solids*, **49**, 1181-1208
[6] Hild F., Denoual C., Forquin P. and Brajer X. (2003), *Computers and Structures*, **81**, 1241-1253
[7] Shenoy V.B. and Kim K.-S. (2003), *Journal of the Mechanics and Physics of Solids*, **51**, 2023-2035
[8] Maiti S., Rangaswamy K. and Geubelle P.H. (2005), *Acta Materialia*, **53**, 823–834
[9] Zhou F., Molinari J-F. and Ramesh K.T. (2006), *International Journal of Fracture*, **139**, 169-196