DYNAMICAL EFFECTS IN THE PROBLEM OF FILM DELAMINATION

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Summary In modern constructions, thin layer coats are often used as protecting or strengthening elements. Deformations of such constructions may cause significant stresses on the interface between the base and the coat because of the difference in their physical mechanical properties, which leads to the destruction or delamination of the cover. Of special interest is strength analysis under dynamical or vibrational impacts because of the possibility of localizing oscillations in a neighborhood of the initial inhomogeneities. In this paper, on the example of the delamination of a string from an elastic substrate, the influence of wave localization on development of defects of the type crack-delamination is analyzed. The model allows us to obtain an approximate analytical solution and to analyze the conditions of growth of delamination zone. A numerical modeling of the problem is performed, and the results of modeling are compared with the approximate analytical solution.

INTRODUCTION

Localization effects of dynamic processes in deformable solids in regions containing inclusions with their own dynamics, were first investigated by Vorovich I. I. and Babeshko V. A. [1]. Most of the known works [2-4] are devoted to the definition of necessary and sufficient conditions for the existence of discrete spectrum for the various spectral problems. The solution of the non-stationary problems, namely, the study of the formation of localized waves in a continuous medium with inclusion-values in the case of unsteady loading is extremely rare.

The question of changing the stress state in the areas of localization of the dynamic process in comparison with the stress state generated by the transmitted and reflected waves is open. This is important for engineering evaluation of contact strength on and acoustic properties of the medium. It should be noted that the definition of the dynamic behavior of a continuous medium with the inclusion, when there is only localized state, greatly simplifies the solution of the complete original Cauchy problem, taking into account the transient wave processes.

The paper contains the solution of two important from the standpoint of engineering applications problems of transient behavior of an elastically deformable-coating the substrate in the presence of a defect in the form of detachment. The solution of the first task allows to find the necessary parameters of the structure, in which the phenomenon of localization leads to a significant increase of stresses in the vicinity of detachment compared with transient wave process. The solution of the second problem describes the growth of the exfoliation. The necessary conditions of the stop of the growth depending on the parameters of the substrate and film and on the loading parameters are obtained.

PROBLEM STATEMENT

As a model of the film on the substrate we will take a string on an elastic foundation with stiffness $k\big[x, l(t)\big]$ which is equal zero on the delamination zone, and outside this region is equal to k_0 . We consider the force loading in the form $P(t)\delta(x - x_p)$ where δ is Dirac delta function. In this case the equation of the form of the Klein-Gordon equation describing the movement of the string on an elastic foundation is given by

$$\begin{aligned} &\rho u_{tt} - T u_{xx} + k\big[x, l(t)\big] u = P(t)\delta(x - x_p), \quad -\infty < x < \infty, \\ &u,\ u_x \to 0, \quad |x| \to \infty. \end{aligned} \tag{1}$$

Here ρ is the length density of the string, u is the vertical displacement of the string, T is the tension of the undisturbed string. We assume that the initial time. For the detachment criterion we take the following deformation criterion: when the displacement of at least one end of the detached part attains a critical value Δ, the delamination zone grows with rate β. The equation describing the growth of the detachment zone has the form

$$\frac{\mathrm{d}l}{\mathrm{d}t} = \frac{\beta}{2}\Big\{H\big[u(x,t)|_{x=l_-(t)} - \Delta\big] + H\big[u(x,t)|_{x=l_+(t)} - \Delta\big]\Big\}. \tag{2}$$

Here, l_- is the coordinate of the left end of the detached fragment, l_+ is the coordinate of the right end of the detached fragment, $H(t)$ is the Heaviside function, is the coefficient determining the growth rate of the delamination.

LOCALIZATION. STATICS AND DYNAMICS

In the case of a fixed zone of delamination $l(t) = l_0$ there is the exact solution of the problem. The latter allows to compare the contribution of the localized solution which corresponds to the discrete spectrum of eigenfrequencies of oscillations ω_j [5] and transient wave process (a continuous spectrum) and to determine the values of and the film parameter ω_b (cutoff frequency), for which there is only one localized mode oscillation, which is formed in delamination zone.

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Localization of waves in the area of film delamination leads to an increase in the amplitude of forced oscillations. Such oscillations in the presence in the system of negligible dissipation can exist very long. It should be noted that there is a range of parameters for which the amplitude of localized oscillations can be much larger than the amplitude of propagating waves. When the parameter $\omega_b l_0/c \to \pi/2$ the growth displacement at the detachment ends is observed. Note that $\omega_b l_0/c$ is the partial frequency of oscillations of a string of length $2l_0$ on elastic foundation with tightly clamped ends.

THE FILM DELAMINATION

We assume that the initial delamination area has dimensions satisfy the conditions $\omega_b l_0/c \to \pi/2$. In this case, for the corresponding spectral problem has a single value of the discrete spectrum, which corresponds to a symmetrical mode, and a continuous spectrum, starting at the cutoff frequency. Localization of the wave process on the defect is due to the symmetric modes, so we assume that the growth zones of delamination is also symmetric. The solution of the original nonlinear problem will be sought using the Bubnov-Galerkin method for a variable interval

$$u(x,t) = v_0(x,l,\omega_0)\, q_0(t), \tag{3}$$

where ω_0 is eigenfrequency, v_0 is the corresponding symmetric localized eigenmode, q_0 is the unknown function (generalized coordinate). Transient processes associated with the propagation of traveling waves are not considered in the present range of parameters. Substituting (3) of the vibrating string equation (1), multiplying the resulting equation with and integrating over , we obtain the following equation for q_0. This equation is obtained under the assumption that changes in the initial eigenfrequency, which determines the shape of the oscillations with $l(t) = l_0$, are small and also it was taken into account the dispersion relation determining the dependence of its own localized frequency $\omega_0(l)$.

Let us consider the oscillations caused by the impulse exciting force $P_0\delta((x)\delta(t)$. Then, for $t < t_1$ the value of detachment zone is constant and determining the generalized force and the generalized mass , we can easily obtain the solution of the equation for $q_0(t)$. Substituting it into (2) we obtain that a necessary condition for the growth of delamination in the form

$$\frac{P_0 \cos\lambda l_0}{M_0(l_0)\,\omega_0} \geq \Delta, \tag{4}$$

where $\lambda = \omega_0(l)/c$, $M_0 = \rho(l + 1/\gamma)$ If the initial conditions are such that inequality (4) holds, then the linear growth of delamination zone at a rate starts at time , which can be approximately determined by the following expression

$$\sin\omega_0 t_1 = \frac{\Delta\,\omega_0 M_0(l_0)}{P_0\cos\lambda l_0}, \quad l(t) = l_0 + \beta\, t\, H(t - t_1)$$

The length of the delamination zone is growing till the point in time $t = t_2$ while the deformation criterion (2) is performed. When the delamination zone length remains constant and equal $l_1 = l(t_2)$. Delamination growth zone resumed at time $t = t_3$, when the amplitude of the film reaches Δ, and so on. Such a stepwise increase of the delamination zone continues until $l(t)$ does not reach a certain critical value there. Indeed, with increasing length of the delamination zone the amplitude of the oscillations of a string decreases and thus there is a moment of time and the corresponding length $l = l_k$, where the condition (9) is not valid and the growth of delamination ceases.

Thus, an approximate analytical solution, built on the basis of the localized mode, describes the stepwise growth of delamination at a rate determined by the parameter β.

CONCLUSIONS

The the conditions for growth and stop of delamination are formulated. It should be noted that it is taken into account only the first symmetric mode shape by the construction of an approximate analytical solution and it is selected the most simple criterion for delamination growth zone, which does not include real rheology in the vicinity of delamination boundaries. However, even the simplified model can explain the observed failure of structures with thin-layer coatings caused by localization of oscillations on various defects at relatively low loads.

References

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