FATIGUE CRACK GROWTH ANALYSIS OF SHORT CRACKS IN CONCRETE

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Summary In the present work, an analytical model has been developed to determine the fatigue crack propagation of short cracks in plain concrete based on the principles of dimensional analysis and self-similarity. The model involves various important parameters influencing the short crack propagation and is able to capture the crack growth phenomenon in the initial stage of crack propagation that is when the length of crack is very small (short crack) as well as in zone I of the crack growth curve. Through a validation study, the predicted results are compared with the experimental trend and found to be in good agreement.

INTRODUCTION

The crack growth curve in a structure when subjected to fatigue loading depicts a sigmoidal pattern. The entire curve can be divided into three distinct regions namely zone I, zone II and zone III as represented in Figure 1, depending on the propagation rate. The most widely accepted fatigue crack propagation law, proposed by Paris et al. is used to compute the crack propagation in zone II of the growth curve. The rate of crack propagation in zone I is very slow when compared to zone II and it is strongly dependent on material micro-structure and stress ratio. A threshold stress intensity factor (K_{th}) exists in zone I below which no detectable crack propagation is observed. Various researchers have shown [1, 2]

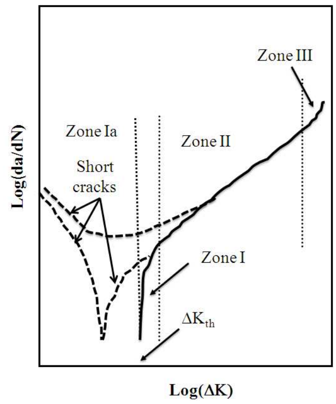

Figure 1. Fatigue crack propagation showing short crack growth

that crack initiation and propagation occurs even below the threshold stress intensity factor. The crack propagation below the threshold is shown in Figure 1, named as zone Ia and called as short cracks. Through extensive experimental studies on metals, it has been observed that short cracks as shown in Figure 1 grow at a faster rate than the longer counterparts and propagates even below the threshold stress intensity factor. The fracture mechanics based life estimation is invariably for long crack data (zone II) and extrapolation for short cracks (increased growth rate) might lead to a non-conservative estimation of life. Therefore, for accurate estimation of fatigue life of structures, all the regimes of crack propagation need to be addressed including the growth of short cracks which occur even below the threshold values.

In the present work, an attempt has been made to understand the short crack growth mechanisms, which is a completely new concept in quasi-brittle materials like concrete. By understanding the experimentally observed high growth rate of short cracks, an analytical model has been developed to estimate the crack extension in the short crack regime using the concepts of dimensional analysis and self-similarity.

ANALYTICAL MODEL FOR CRACK GROWTH PREDICTION OF SHORT CRACKS

In the present context, for quasi brittle materials like concrete, short cracks are defined based on the micro-structural length scale in concrete. In plain concrete, micro-structural length scale correspond to average size/maximum size of the aggregate. Considering the ratio of structural size to maximum grain size as the characterizing parameter, definition of short cracks can be proposed for concrete. Also, this ratio is strongly dependent on the failure mechanism of the structure and can be used as a parameter for short crack. These cracks can be defined as having a size of $\alpha(D/d_{max})$. The value of constant α can be set based on the experimental data. Crack sizes $\leq \alpha(D/d_{max})$ are considered to be short for concrete and has been observed experimentally to exhibit a higher growth rate than its longer counter part.

The development of the analytical model is based on the concept of dimensional analysis and self-similarity. By knowing the governing parameters of a physical phenomenon, the functional dependencies between the various quantities can be obtained through different assumptions of self-similarity [3]. Here, self-similarity means the phenomenon reproduces

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itself at different time and space scale. In the present study, crack growth phenomenon is found to be dependent on several quantities namely - loading parameter ΔG_I , the tensile strength (σ_t), threshold value of fracture energy (ΔG_{th}), crack length (a), structural size (D), maximum aggregate size (d_{max}) and stress ratio ($1 - R$). The proposed model for rate of crack propagation is given by

$$\frac{da}{dN} = \Delta G_I^{\gamma_1} \Delta G_{th}^{1-\gamma_1-\gamma_2} \sigma_t^{\gamma_2 - 1} a^{\gamma_2} (1 - R)^{\gamma_3} \Phi_3 \tag{1}$$

The coefficients γ_1, γ_2 and γ_3 introduced in the formulation stage can not be determined from the dimensional analysis. These coefficients have to be determined thorough a calibration using experimental data and regression analysis.

CALIBRATION AND VALIDATION OF THE PROPOSED MODEL

In the calibration study, the coefficients in Equation 1 are determined using the available experimental data. For this the experimental results of Shah [4] has been used. Although it is generally assumed that the crack propagation is negligible when the alternating stress intensity factor range is less than the threshold value, the experimental results of Shah [4] show otherwise wherein cracks of small size is observed to propagate at a faster rate even below the ΔK_{th}. The proposed model has been calibrated to predict this crack propagation in zone Ia and zone I of the growth curve shown in Figure 1. To calibrate the model, experimental data and results of small size specimen is considered and the coefficients are determined through an optimization process using the principle of least squares. The best predicted values for the coefficients γ_1, γ_2 and γ_3 in zone Ia are obtained as -3.61, -0.067 and -0.038 respectively. In the experimental data, although short cracks have shown an increased growth rate, deceleration occurs up to the threshold value of stress intensity factor and hence the calibrated coefficients for γ_1 and γ_2 are negative. After the threshold point, in zone I, acceleration in crack propagation occurs at a slower rate. To predict the crack growth in zone I the calibrated values of γ_1, γ_2 are 0.08 and 0.01 respectively. It is to be noted that Φ_3 is a function of structural size i.e. d_{max}/D. To determine the value of Φ_3 for structural sizes other than used for the calibration study, the relationship between Π_3 and and Φ_3 shown in Figure 2 can be referred. Further, using the computed values of the coefficients, the proposed model has been validated with the

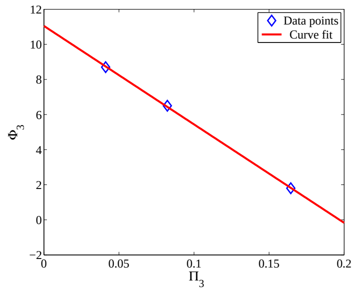

Figure 2. Relationship between Φ_3 and the nondimensional parameter (Π_3)

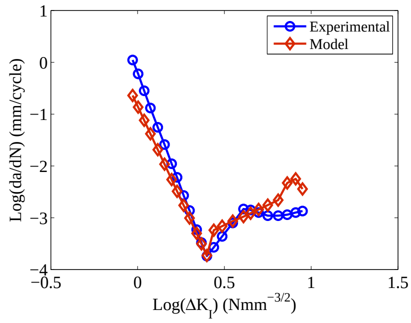

Figure 3. Crack growth in large sized beam specimen

experimental results [4] that are not used in the calibration study. Figure 3 shows the plot of crack growth rate for both model predictions and experimental data considering medium sized specimen. The proposed model is observed to agree well with the experimental data.

CONCLUSIONS

Research on short cracks in concrete like quasi-brittle materials is a new concept and has been addressed in the present work. Along with the definition of short cracks in concrete, an analytical model has been proposed to capture the crack propagation of short cracks (zone Ia) as well as the cracks falling in zone I region of the growth curve considering various important crack growth influencing parameters such as crack length, micro-structural parameter (d_{max}/D), threshold fracture energy, crack driving energy and tensile strength. The model is derived using the concepts of dimensional analysis and self-similarity thereby avoiding the empirical nature. The model predictions have been compared with the experimental results of plain concrete beams available in the literature and is seen to agree well.

References

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