MECHANICS OF EDGE CRACKS DUE TO A PHASE TRANSFORMATION

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Summary We examine the growth of edge cracks in a brittle material due to a solid-solid phase transformation. We show through a combination of analytical and computational means that equi-spaced parallel cracks form at a critical spacing dependent on the transformation strain, all cracks grow up to the transformation front, and continue to grow uniformly as the transformation progresses. We show that our results are in agreement with experimental observations, and discuss implications on various applications.

INTRODUCTION

Solid-solid phase transitions arise in a number of important technological applications ranging from active materials like shape-memory alloys and ferroelectrics, materials of interest to energy applications such as lithium-ion batteries and solid oxide fuel cells to alloys like steels and superalloys. These transformations involve a change of crystal structure and consequently internal stress. This internal stress can lead to fracture when the material is brittle, and the resulting cracks could continue to grow as the phase boundary proceeds into the sample in a directional nature.

Directional crack growth has been studied in a variety of situations including thermal shock, basaltic column formation and solidification where inhomogeneous shrinkage or expansion due to a temperature or concentration gradient results in stresses. One can distinguish between two situations, one where one has a network of cracks due biaxial tension and another where one has largely parallel edge cracks due to uniaxial tension. The latter can arise either due to the nature of the mismatch or due to geometry leading to plane stress. The analysis of the directional growth of a set of edge cracks due to a temperature gradient was carried out in detail by Nemat-Nasser[2] and Bazant [1]. By assuming a certain smooth profile for the temperature gradient, a set of parallel edge cracks was analyzed. It was found that at a certain depth of the temperature profile, this mode of growth is susceptible to a period doubling instability where every alternate crack arrests.

In the current work we investigate directional cracking due to a solid-solid transformation. In contrast to the situation of thermal and composition gradient that is the subject of much of the literature, phase transformations involve a sharp change or front across which the stress-free strain changes. We show that this dramatically changes the nature of crack growth. We focus specifically on edge cracks and show that when the transformation strain variation is sharp, parallel cracks form at a critical spacing dependent on the transformation strain and all cracks grow up to the transformation front. Further, they all progress with the transformation front as the transformation progresses. In particular, no period doubling or other instability is observed. We show that our results are in agreement with experimental observations [3] and discuss implications for various applications.

PARALLEL EDGE CRACKS

A solid-solid phase transition is characterized by a transformation strain. Depending on the transformation strain and geometry, we can have three cases: no internal stress, tensile internal stress in two principal directions and tensile stress in only one principal direction. In this work, we consider the last case, and this can be reduced to a two-dimensional problem.

We consider a set of parallel equi-spaced edge cracks in a half-plane growing in the wake of a phase boundary as it moves quasi-statically from the edge to the interior, see Figure 1 (left). We assume that the elastic moduli are identical on either side of the phase boundary. Let the transformation strain be given by $\epsilon^* = \mathrm{Diag}[-2\epsilon_0, \epsilon_0]$. We show that it is sufficient to consider a straight phase boundary to describe its interaction with the cracks, and therefore limit our attention to this situation. The stress field is evaluated using superposition. The two sub-problems for the superposition are explained in the Figure 1. In the first subproblem (case-A) we assume that the cracks are absent and only the phase boundary is present. The stress state we obtain in this case is the piece wise homogeneous state like in a thin film case. In the second case (case-B) we assume that the phase boundary is absent and the crack faces are loaded with tractions obtained from case A, $\mathbf{t} = -\sigma^{(A)}.\hat{e}_2 = -\sigma_o \hat{e}_2 = -E\epsilon_o \hat{e}_2$.

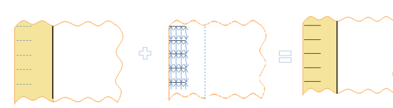

Figure 1: Setup and superposition

We adopt the distributed dislocation methodology as outlined in [2] to determine the stress state in case-B and the Mode-I stress intensity factors (SIF) . The procedure is described here for the case with alternating crack lengths h_1 and h_2 with a spacing b between them. The cracks are loaded by a uniform stress σ_o, when they are within the range $0 < x < L$. Due to the periodicity, we consider only a strip of width $2b$. We introduce unknown dislocation distributions $D_(1)$ and $D_(2)$ on the two crack faces, and write down the traction boundary conditions on the crack faces. This gives two integral equations for the unknown distributions, and these integral eqauations are solved numerically by using Gauss-Chebyshev quadrature after taking the crack-tip singularity into account. Once $D_1(t)$ and $D_2(t)$ are determined, they can be used to determine the full stress field and the SIFs, K_1 and K_2 in the problem.

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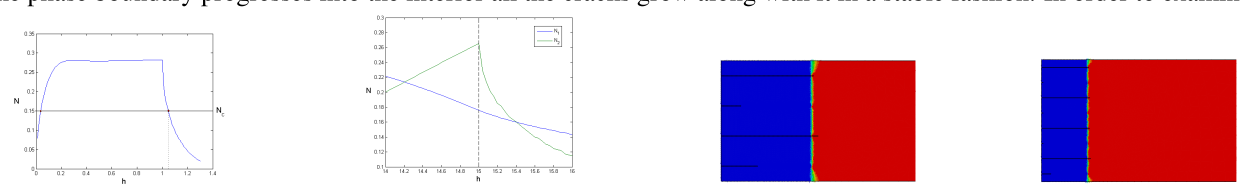
STABILITY ANALYSIS OF A SYSTEM OF CRACKS

The stability of a system of cracks in a brittle, homogenous, elastic medium is analyzed by considering the variation of the potential of the system, $\Pi = W(a_1, a_2, ..., L, \nabla u) + \sum_{i=1}^{M} 2\gamma a_i$, at a fixed load level L, with respect to the crack lengths $[a_1, a_2, ...]$ following [1]. The first variation of the potential leads to the well known Griffith condition : $K_i = K_c \quad \text{for} \quad \delta a_i > 0 \quad \text{and} \quad K_i = 0 \quad \text{for} \quad \delta a_i < 0$. The stability of the equlibrium is determined by examining the positive definiteness of the second variation with respect to a perturbation satisfying certain admissibility conditions. In the present case we examine a set of periodic edge cracks with two cracks unit cell. For the mode of homogenous crack growth (all cracks growing equally maintaining equal length), the stability criterion reduces to $\dfrac{\partial K_1}{\partial a_1} = \dfrac{\partial K_2}{\partial a_2} < 0$, see [1]. This mode reaches a critical point for some value of value of $L = L_c$ when the derivatives become zero. At this point the homogenous growth mode is no longer stable and other modes of crack growth need to be examined for stability. We examine the stability with respect to the period-doubling mode, where every other crack stops growing and the rest of the cracks continue to grow, $\delta a_1 = 0, \ \delta a_2 > 0$. In this situation the stability condition reduces to: $K_1 < K_c \qquad K_2 = K_c \qquad \frac{\partial K_2}{\partial a_2} < 0$.

RESULTS AND DISCUSSION

Following the procedure briefly outlined above we arrive at the two plots shown below. Plot (2a) shows the variation of $N = \frac{K}{\sigma_o\sqrt{\pi b}}$ with crack length h for homogenous crack growth mode. The horizontal dotted line represents the fracture toughness K_c. This plot tells us that for a given crack spacing, the homogenous crack growth mode has two possible equilibrium ($K_1 = K_2 = K_c$) lengths. The stability condition for this mode of growth ($\frac{\partial K}{\partial a} < 0$) is however only met by the point on the right, for lengths of cracks crossing over the interface. Figure (2b) shows the variation of N_1andN_2 with, $h_1 = h_{eq}$ being held at its equilibrium value, and letting h_2 vary. This plot helps us examine the stability of the configuration with respect to the period doubling stability. Since the derivative $\left.\dfrac{\partial K_2}{\partial h_2}\right|_{h_1,L}$ which decides the stability, is clearly negative we can conclude that at this value of L the equal crack configuration is not susceptible to the period doubling instability. Other values of L were also examined and a similar conclusion was arrived at. So this analysis leads us to believe that during a phase transformation with a sharp phase boundary, the cracks which get nucleated and grow, propagate all the way to the interface and cross over it. As the phase boundary progresses into the interior all the cracks grow along with it in a stable fashion. In order to examine the appearance of other modes of instability we resort to Finite Element Simulations. We simulate unit cells containing various number of nucleii and observe the evolution of cracks as the phase boundary propagates.



(a) Equilibrium states for homogenous growth mode.

(b) Variation of N_1 and N_2 with h_2. The dotted line indicates the interface.

(c) Four cracks in unit cell

(d) Eight cracks in unit cell

Figure 2

The simulations reveal that not all nucleii grow into cracks. The cracks which develop, grow at a constant spacing along with the interface. The cracks cross over the interface. The spacing between the cracks decreases with increase in the stress caused due to the transformations strain. The patterns do not exhibit any period doubling instability. However some of the nucleated cracks stop intermediately in order to even out the spacing.

CONCLUSIONS

The present work examines the problem of edge crack growth due to a Martensitic phase transformation. The prediction is that, after nucleation the cracks would grow all the way to the interface and cross over and continue to grow as the phase boundary traverses into the interior. The spacing between the cracks is determined by the level of the transformation strain, the cracks being closer with higher levels of stress developed due to the transformation.

References

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