DESTRUCTION OF THIN FILMS WITH A DAMAGED SUBSTRATE AS A RESULT OF WAVES LOCALIZATION .

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Summary
The behavior of a thin film with a damage substrate under strike like loading is modeled as a behavior of a string, an elastic foundation of which is assumed to be imperfect, and which coefficient depends on a damage function of a substrate.The problem of localized modes existence reduced to the solution of a Schrodinger equation. The asymptotic solution leads to the following conclusion:there is a possibility of a wave localization in the region of the damage substrate. There are possible three scenario of the damage function behavior: a monotone growth,a piecewise like constant growth,and no growth.

STATEMENT OF THE PROBLEM

Some types of a laminated safety glass consists of an interlayer surrounded by two thin films (see in [1]). An optimal level of adhesion is then required for the safety glass to absorb enough of the impact energy to prevent projectile penetration and delamination. Such considerations have motivated us to study the behavior of such structures undergoing shock like loading and introduce the mathematical model of the structure behavior.The structure under consideration is modeled as a string, an elastic foundation of which is assumed to be imperfect, and which coefficient depends on a damage function of a substrate.The imperfection of an elastic foundation is modeled by a damage function for which the evolution equation is introduced.The governing equation is as follows:

$$\gamma u_{xx} - K(n)u - \rho_0 u_{tt} = A\delta_\epsilon(x)\delta(t)., \quad x \in (-\infty, +\infty), \quad t > 0 \tag{1}$$

where u is a displacement, Q is an external force.Here δ_ϵ is a smoothed δ function, we can define it, for example, as a Gaussian peak $\delta_\epsilon(x) = \epsilon^{-1}(2\pi)^{-1/2}\exp(-\frac{x^2}{2\epsilon^2})$. Here $K[u]$ is a complicated functional of u that involves a damage function n. The quantity K depends on n via the following relations

$$K(n) = \mu(n)G(n), \quad \mu(n) = \frac{k_0}{k_0 + G(n)} \tag{2}$$

where $k_0 > 0$ is a constant, $G(n) = G_0(1-n), \quad 0 \le n \le 1$; $G(n) = 0, \quad n > 1$, Here G_0 is a constant. The time evolution of the damage function $n(x,t)$ is defined by the differential equation

$$\frac{\partial n}{\partial t} = \beta H(\mu(n)|u| - \Delta)(1-n), \tag{3}$$

where $\beta, \Delta > 0$ are positive constants, and H is the Heaviside step function. We assume that Q is a fast decreasing in $|x|$ smooth function and we set the boundary and initial conditions $u(x,t) \to 0 \quad (|x| \to \infty); u(x,0) = 0, \quad u_t(x,0) = 0, \quad x \in (-\infty, +\infty); n(x,0) = n_0(x), \quad x \in (-\infty, +\infty)$ where n_0 also is a fast decreasing in $|x|$ smooth function.If $x \in [-h,h]$, we set the Dirichlet boundary condition $u(h,t) = u(-h,t) = 0$. There are possible alternative versions of eq. (3) for the damage function: $\frac{\partial n}{\partial t} = \beta H(|u_x(x,t)| - \epsilon_*)(1-n)$, if we assume that material is destroyed at point x at the moment t if a deformation $\epsilon(x,t)$ attained to some level ϵ_*:$\epsilon(x,t) = |u_x(x,t)| = \epsilon_*$, or another expression: $\frac{\partial n}{\partial t} = \beta H(v(x,t) - \sigma_*)(1-n)$ which valid for the "incubation time" criterion.That criterion has the form:$\tau_0^{-1}\int_{t-\tau_0}^{t} u_x(x,s)ds| = v(x,t) = \sigma_*$ (for details, see [2]). Also we assume that our material reached the full destruction at a point x at the moment t if the damage parameter $n(x,t)$ attain, at this moment, to some critical level n_* close to 1(the elastic foundation coefficient vanished and the delamintion occurs).Let us introduce the front of full destruction $L_F(t)$ by the relation $n_* = n(L_F(t),t)$, where n_* is a critical damage level. The $L_F(t)$ is an increasing time function in general case when $u(x,0) \ne 0$.
If β is small, then $dn/dt << 1$ and the coefficient K is a function of the slow time $\tau = \beta t$. Therefore,we can use a quasi-stationary approximation by the two-time scales perturbation method. We assume that $\beta << 1$ and $K(n)$ is a function of the slow time $\tau = \beta t$. For each fixed τ, there occurs a Schrodinger operator associated with this problem,

$$\mathcal{H}\Psi = \frac{d^2\Psi}{dx^2} - \Phi(x,\tau)\Psi = E\Psi, \quad \Phi = \gamma^{-1}(K(n(x,\tau)) - \bar{k}_0). \tag{4}$$

The eigenfunctions Ψ may be localized ones, when $\Psi = \Psi_j(x,\tau),\ E = \gamma^{-1}(\bar{k}_0 - \rho_0^{-1}\omega_j^2)$, where $j = 1,...,N(\tau)$, and may belong to the continuous spectrum, when $E = -k^2 \le 0$ and $\Psi(x,k,\tau)$ have asymptotic $\exp(ikx)$ as $x \to +\infty$ and $a(k)\exp(ikx) + b(k)\exp(-ikx)$ as $x \to -\infty$, where $a(k), b(k)$ are some coefficients.For the continuous spectrum it is useful to introduce the frequency ω by $E(k) = \gamma^{-1}(k_0 - \rho_0\omega^2(k))$. The potential Φ is non-positive

$$\Phi = -\frac{k_0^2 G_0 n}{\gamma(k_0 + G_0)(k_0 + G_0(1-n))}. \tag{5}$$

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The analysis shows that this leads to the following conclusions:for all times there always exists a localized eigenfunction and, thus, $N(\tau) \geq 1$; the mode number N also can depend on τ. Since n is an increasing time function and $K(n)$ is a decreasing in n function (see above), the module of our potential $|\Phi(x,\tau)|$ is an increasing function of τ, therefore the mode number always increases in time. Notice that the continuous spectrum lies within $E(k) \in (-\infty, 0), \quad \omega > \sqrt{\bar{k}_0 \rho_0^{-1}}$ and the discrete spectrum lies within $E_j \in (-\max|\Phi|, 0))$. We thus obtain an asymptotic solution consisting of the two following contributions:

$$u(x,t,\tau) = W(x,t,\tau) + V(x,t,\tau), \tag{6}$$

where

$$V(x,t,\tau) = -\rho_0^{-1} \sum_{j=1}^{N(\tau)} a_j (c\omega_j(0))^{-1} \sin(S_j(t)) \Psi_j(x,\tau) \tag{7}$$

$$W(x,t,\tau) = -\rho_0^{-1} \int_0^\infty a_k(\tau)\omega(k,0)^{-1}\Psi(x,k,\tau)\sin S(k,t)dk, \tag{8}$$

where

$$a_j(\tau) = \int_{-\infty}^{\infty} \delta_\epsilon \Psi_j(x,\tau)dx, \quad a_k(\tau) = \int_{-\infty}^{\infty} \delta_\epsilon \Psi(x,k,\tau)dx. \tag{9}$$

For S_j and $S(k,t)$ we can use the standard WKB approximation that gives $\frac{dS_j}{dt} = c\omega_j(\beta t), \quad \frac{dS(k,t)}{dt} = c\omega(k,\beta t)$. The sequential analysis was made for particular types of the initial damage parameter distribution (rectangle and Gaussian) and leads to the following results. For the case when $k_0 << 1$ but $G_0 = O(1)$ or $G_0 << 1$ and $k_0 = O(1)$ one can get the following asymptotic for the displacement $u(x,t)$:

$$u(x,t) = A\rho_0^{-1}[(2c)^{-1}(H_\epsilon(x-ct) - H_\epsilon(x+ct)) - \omega_0^{-1}\Psi(x_0)\Psi(x)\sin(\omega_0 t)]. \tag{10}$$

Here H_ϵ is a smoothed step function, A is the force amplitude,c is the wave propagation speed, $c^2 = \gamma/\rho_0$, Ψ is defined by

$$\Psi(x) = C\cos((U_0 - E)/\gamma)^{1/2}x), \quad |x| < l, \tag{11}$$

$$\Psi(x) = C\cos((U_0 - E)/\gamma)^{1/2}l)\exp(-(E/\gamma)^{1/2}|x-l|) \quad |x| > l. \tag{12}$$

Even in this simplest situation, the numerical simulations show a complicated destruction front dynamics. The second term determine the oscillating character of the front motion, but for $t >> 1$ the wave term is larger and oscillations become smaller and smaller.On the contrary, for $k_0 >> 1$ the wave contribution is small. Using obtained solution it was possible to calculate the time when the destruction process start and also the time which is necessary for a full destruction.We have two qualitatively different dynamics of the front. If the localized mode amplitude is small with respect to the amplitude of the wave term the dynamics of the front is linear. If the localized mode amplitude has the same order then the wave term amplitude, $|v_x|$ may be smaller then the critical value $\tau_0\epsilon^*$ for some t, and bigger for other t. Therefore, we have alternation of time intervals,where L_F increases, and time intervals where $L_F(t)$ is a constant. We can observe a transition from oscillating dynamics to a wave dynamics with a linear growth when the front leaves the zone where the localized mode amplitude is not small. Finally, for large times, the front moves in a linear manner, and for very large times the front stops since the amplitude of the wave contribution slowly decreases as $t^{-1/2}$. The time when the front stop can be estimated roughly by an asymptotic for the wave contribution that gives: $T_{stop} \approx A\epsilon^2\bar{k}_0\gamma^{-1}\sqrt{\rho_0}$. Moreover, there is possible a transition from oscillating front dynamics with stop intervals to a monotone linear if the zones of the strike force application, and the location of the localized mode, are different. Then, when one of the fronts passes the localized mode concentration zone, we observe a transition from the linear dynamics to an oscillating dynamics, and after that we again can observe a linear dynamics and finally the front stops.

CONCLUSIONS

Main effects are as follows: the damage zone in substrate can cause the non-stationary wave localization in it;there are possible transitions between different types of the front destruction dynamics. There are possible three scenario of $n(x,t)$ behavior: a monotone growth of n;a piecewise like constant growth of n (when n increases during some intervals and n is constant between these growth intervals);no growth of n.

References

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[2] Petrov Y.,Morozov N.,Smirnov V.,Structural macromechanics approach in dynamics of fracture,*Fatigue Frac. Engng Mater Struct***26**:363-372,2003.