STRUCTURAL-TEMPORAL APPROACH FOR MULTI-SCALE FRACTURE DYNAMICS

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Summary Structural-temporal theory for the analysis of the multi-scale nature of dynamic fracture based on the notion of a spatial-temporal fracture cell for different scale levels is presented. It was shown that corresponding incubation time based approach can be an effective instrument for both modelling principal dynamic fracture effects and establishing interconnections between dynamic strength properties on different scales.

Introduction

Dynamic fracture of quasi-brittle materials is determined by processes at several different scale levels. In order to model the fracture of such materials, it is necessary to account for all the rupture processes that contribute to the overall fracture process. The most common difficulty in predicting the dynamic fracture of typical quasi-brittle materials (concrete, rocks) is the lack of adequate experimental data on the material properties at the relevant scale level. The tests on these materials are performed on laboratory scale samples, while the size of most structures made of them ranges from tens of meters (concrete structures) to kilometers (geological objects). Thus it is necessary to predict the behavior of the material on the large size level using the properties measured on a smaller scale level. In order to do so certain assumptions have to be made consistent with the knowledge about the dependency of the fracture properties on the process scale level. Should these assumptions be confirmed by experiment at the laboratory scale, then it will be possible to predict the fracture properties on the "next" scale level to the laboratory scale. It is even conceivable that for some quasi-brittle materials it will be possible to predict fracture on any scale level from the properties measured on the laboratory scale level.

Structural-temporal approach

In this paper we are trying to sum up our knowledge and experience in experimental, theoretical and numerical investigations of multiscale fracture and apply this to model fracture of quasi-brittle heterogeneous materials on different scale levels. The idea about interconnection of fracture scale levels is originating from classical concept of "fracture linear size" developed into a "spatio-temporal cell" in [1-3]. Corresponding incubation time fracture criterion, originally proposed to predict crack initiation in dynamic conditions, was used in numerous applications [4]. This criterion for fracture at a point x, at time t, reads as:

$$\frac{1}{\tau}\int_{t-\tau}^{t}\frac{1}{d}\int_{x-d}^{x}\sigma(x',t')\,dx'dt' \geq \sigma_c\,. \tag{1}$$

where τ is the incubation time of a fracture process (or fracture microstructural time) – a parameter characterizing the response of the material to applied dynamic loads (i.e. τ is constant for a given material and does not depend on problem geometry, the way a load is applied, the shape of a load pulse or its amplitude); *d* is the characteristic size of a fracture process zone and is constant for the given material and chosen spatial scale. σ is normal stress at a point, changing with time and σ_c is its critical value (ultimate stress or critical tensile stress evaluated in quasistatic conditions); *x'* and *t'* are local coordinate and time. We assume

$$d = \frac{2}{\pi}\frac{K_{Ic}^{2}}{\sigma_c^{2}} \tag{2}$$

Condition (2) arises from the requirement that (1) is equivalent to Irwin's criterion ($K_I(t) \geq K_{Ic}$), in quasistatic conditions ($t \to \infty$). This means that a certain size typical for fractured material appears. This size should be associated with a size of a failure cell on the current spatial scale – all rupture sized essentially less than d cannot be called fracture on the current scale level.

Thus, by introduction of *τ* and *d* time-spatial domain is "quantized". Once material and scale one is working on are chosen, τ gives a time, such, that energy, accumulated during this time can be released by rupture of the cell that accumulated it. Linear size *d* assigns dimensions for the cell. Introduction of temporal and spatial domain discretization is a very important step. To our belief, a correct description of high loading rate effects is not possible if this time-spatial discreteness is not accounted somehow. Advantage of incubation time approach is that one can stay within the framework of continuum linear elasticity, utilizing all the consequent advantages and accounting discreetness of the problem only inside critical fracture condition [3,4].

Interconnection of rupture processes on different scale levels

Consider fracture process that is determined and controlled by the incubation time criterion. Assume that the set of material parameters of the criterion σ_c, d, τ (or σ_c, K_{Ic}, τ) is associated with the given scale level. We suppose that any given scale level is characterized by two characteristic lengths representing its lower and upper limits. The first one is lower boundary: $d \cong \frac{2}{\pi}\frac{K_{Ic}^{2}}{\sigma_c^{2}}$. Objects sized essentially less than the lower boundary cannot display fracture on the given scale level. Therefore, test samples smaller in size than this bound cannot be used to determine experimentally the dynamic fracture parameters at this scale level. The second one is upper boundary: $D \cong c\,\tau$ where c is the speed of the energy transport. This sets an upper limit on the volume of material in which the total energy accumulated during the incubation period is able to produce fracture. Test samples larger in size than this bound cannot be used to determine experimentally the dynamic fracture parameters at this scale level. Permissible characteristic sizes L of tested specimens for the given scale level (object sizes for which any fracture theory based on strength properties (σ_c, K_{Ic}, τ) measured on this scale level is valid) should stay within approximate range: $d \le L \le D$. Thereby it is supposed further that any i-th scale level is characterized by a pair of linear sizes {d_i, D_i} and the admissible range of test specimen size is

$$d_i \le L \le D_i \,. \tag{3}$$

We now choose

$$d_{i+1} = D_i \tag{4}$$

so that

$$\tau_i = \frac{d_{i+1}}{c} = \frac{D_i}{c} \,. \tag{5}$$

Thus, knowing strength properties (including fracture incubation time) on some scale level it is possible to estimate the upper boundary for smaller scale level, the lower boundary for the larger scale level and the incubation time for the smaller scale level. Modified incubation time fracture criterion is giving a possibility to establish interconnections between fractures on different scale levels. It also provides a possibility to assign fracture scale level for different experiments on the same material.

Applications

The problem of the experimental determination of fracture parameters at a particular laboratory scale level and their possible interconnections with higher and lower scale levels is also discussed. It is shown that these interconnections can make possible the prediction of fracture parameters on a higher (real) scale level based on the test data obtained on a lower (laboratory) scale. This possibility is of extreme importance for many applications where the possibility to evaluate material strength properties on real structure scale level does not exist (ex. geological objects, big concrete structures, trunk pipelines, etc.). Some applied problems on the basis of the abovementioned ideas were analyzed including modeling of propagation of dynamic crack in trunk gas pipeline [5]. Application of the structural-temporal approach to this and some other problems showed good corroboration with experimental observations. Some possibilities of generalization of multi-scale incubation time approach to problems of non-linear fracture mechanics (involving large scale yielding and big process zones) are also discussed.

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