MULTIFISSURATION AND DEBONDING OF THIN FILMS. ANALYTIC AND NUMERIC 1D SOLUTIONS VIA A VARIATIONAL APPROACH

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Summary We present a 1D variational model for multifissuration and debonding of thin films. Adopting Griffith-like surface energies and a reduced membrane model for the film, we recover typical and commonly observed features of periodic fissuration and edge debonding via an energy minimization argument. We construct time-continuous evolutions under increasing loads, presenting and comparing the analytic outcome with the results of a finite element implementation of a regularized variational formulation, approximated in the sense of Γ-convergence.

Thin films systems are more and more present in engineering applications: flexible OLED displays, surface coatings, thermal barriers are just to name a few. They are characterized by the presence of a thin layer bonded to a stiff substrate, the layers having very different mechanical properties. During assembly and operating conditions, tensile stresses develop eventually leading to failure with a complex coupling of cracking and debonding. The emerging phenomenology is quite complex: arrays of cracks, nonlinear interaction between cracks and debonding, stop-and-go behavior for straight cracks and structured geometrically complex crack patterns. See [4] for an in depth review of failure modes in the framework of linear fracture mechanics. In such domain, the a priori knowledge of the crack path is necessary.

We free ourselves of this assumption adopting the variational approach to fracture mechanics [3]. We tackle the challenging issue of prediction, characterization and numeric reproduction of these complex phenomena. We treat transverse cracks and debonded regions as free surfaces allowed to appear anywhere at anytime based on an energy optimality principle. A cross section of the 3D model system is sketched in Figure 1 (left): a thin film is bonded to a stiff substrate by

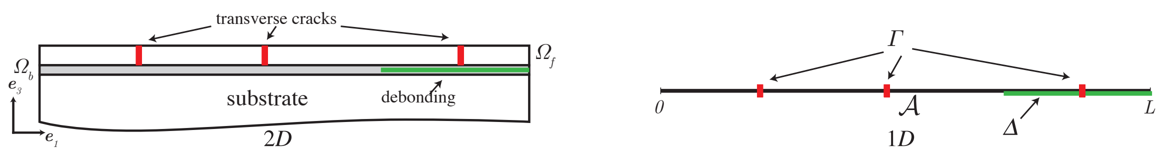

Figure 1. A cross section of the 3D thin film system and the equivalent 1D model: bar on elastic foundation. Transverse fractures are red, debonded areas are green

the means of a thin bonding layer. As it is customary in literature [6], the system is reduced to a 1D equivalent model of a bar on an elastic foundation, Figure 1 (right). Transverse cracks are noted Γ and debonded regions, represented by interfacial cracks in bonding layer, are noted Δ. The displacement field is of the form $\mathbf{u}(x) = u(x)\mathbf{e}_1$, the film is left free at the boundaries. The system is loaded by an inelastic imposed strain $\bar{\varepsilon}_t(x)$, *e. g.* of thermal nature, and by an imposed displacement of the substrate $\bar{u}_t(x)$. Assuming the bar is linearly elastic, the potential energy of the system is given by:

$$\mathcal{P}_t(u, \Gamma, \Delta) := \int_{\mathcal{A}\setminus\Gamma} \frac{1}{2} K \left(u'(x) - \bar{\varepsilon}_t(x)\right)^2 \, dx + \int_{\mathcal{A}\setminus\Delta} \frac{1}{2} k \left(u(x) - \bar{u}_t(x)\right)^2 \, dx \tag{1}$$

The first term is due to the in-plane deformation of the film and the second accounts for the interaction with the substrate. Two constitutive constants appear, namely K and k, representing the film's and bonding layer's stiffnesses.

In order to allow for transverse cracking and debonding, we adopt a Griffith-type fracture energy of the type:

$$\mathcal{S}(\Gamma, \Delta) := G_\Gamma \#(\Gamma) + G_\Delta \mathcal{L}(\Delta) \tag{2}$$

where $\#(\Gamma)$ is the number of cracks, $\mathcal{L}(\Delta)$ the length of the debonded domain, and G_Γ, G_Δ two material constants. The total energy of the system is $\mathcal{E}_t(u, \Gamma, \Delta) = \mathcal{P}_t(u, \Gamma, \Delta) + \mathcal{S}(\Gamma, \Delta)$.

ANALYTIC SOLUTIONS

The qualitative properties of the cracked states can be inferred from the study of the static problem *i. e.* that of finding energy minimizers for given a load. The minimization criterion is *global*, as customary with Griffith-type energies. It is expressed by the following variational inequality:

$$\mathcal{E}_t(u_t, \Gamma_t, \Delta_t) \le \mathcal{E}_t(u, \Gamma, \Delta), \quad \forall u \in \mathbb{H}^1(\Gamma_t), \quad \forall \Gamma \in \mathcal{A}, \forall \Delta \subseteq \mathcal{A}. \tag{3}$$

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From Equation (3), taking smooth and inner variations of the energy we infer the following properties of energy minimizing states. For a given number of cracks n, the optimal arrangement is that of an equally distributed array cutting the film in $n+1$ parts of length L/n. Debonding responds to a pointwise criterion based on the value of $|u(x)|$. It necessarily starts from the boundaries of each of the parts. For a given load, the number of equidistributed transverse cracks and the size of the bonded domain for each of the parts is analytically computed.

The solution of the static problem is the starting point for solving the complete quasi-static time-continuous evolution. As it is well established for the evolution of rate-independent systems [5], we require that the quasi-static evolution satisfies irreversibility, energy balance and unilateral stability. We establish the following results. The number of transverse cracks has to be of the form $n_k = 2^k$, $k \in \mathbb{N}$, *i. e.* transverse cracks appear as a cascade of k simultaneous bisections. Critic loads are computed in closed form. The size of the bonded domain is computed as a function of the load. The complete coupled evolution necessarily consists in a cascade of k transverse fracture phenomena, resulting in 2^k cracks, then debonding. A phase diagram can be constructed, giving the number k of cracking stages the film will undergo before debonding. The complete time-continuous quasi-static evolution $t \mapsto (u_t, \Gamma_t, \Delta_t)$ is retrieved in closed form. We remark that the fundamental properties of the cracked states are retrieved as results via a variational principle instead of being postulated a priori, as it is common practice in literature.

NUMERICAL EXPERIMENTS

Following [2], the numerical implementation is based on the variational approximation of the energy $\mathcal{E}_t(u, \Gamma, \Delta)$ with the regularized functional $\tilde{\mathcal{E}}_{t,\eta}(u, \alpha, \delta)$. It reads as follows:

$$\tilde{\mathcal{E}}_{t,\eta}(u,\alpha,\delta) := \int_{\mathcal{A}} \frac{1}{2}K(1-\alpha(x))^2 \left(u'(x) - \bar{\varepsilon}_t\right)^2 + \frac{1}{2}k\, u(x)^2(1-\delta(x)) + G_\Delta\, \delta(x) + \frac{3}{8}G_\Gamma \left(\frac{\alpha(x)}{\eta} + \eta\alpha'(x)^2\right) dx \quad (4)$$

where $\alpha(x) : x \in \mathcal{A} \mapsto [0,1]$ is the *smeared* representation of the cracks, taking the value 1 along the cracks and 0 away from them. The debonded region is represented by $\delta(x) : x \in \mathcal{A} \mapsto [0,1]$, the characteristic function of the domain Δ and the regularization parameter is η. The approximation is in the sense of Γ-convergence [1], in particular global minimizers of $\tilde{\mathcal{E}}_{t,\eta}$ converge to those of $\mathcal{E}_t$ as $\eta \searrow 0$. The minimization problem for (4) is non-convex and stiff, we take advantage of the convexity with respect to the single variables and perform alternate minimizations until convergence. Formally, the energy (4) is equivalent to a damage functional with η as "internal length". The numerical experiments performed verify the general results obtained in the preliminary analytic study. The peculiar characteristics of the cascade of cracks, boundary debonding, the coupled behavior and the energy evolutions are well captured by the numeric results. In Figure 2 we show a plot of $\alpha(x), \delta(x)$ and the associated elastic, debonding, transverse crack and total energies, for a film split in four parts by three equally spaced transverse cracks. Each of the parts has undergone partial debonding.

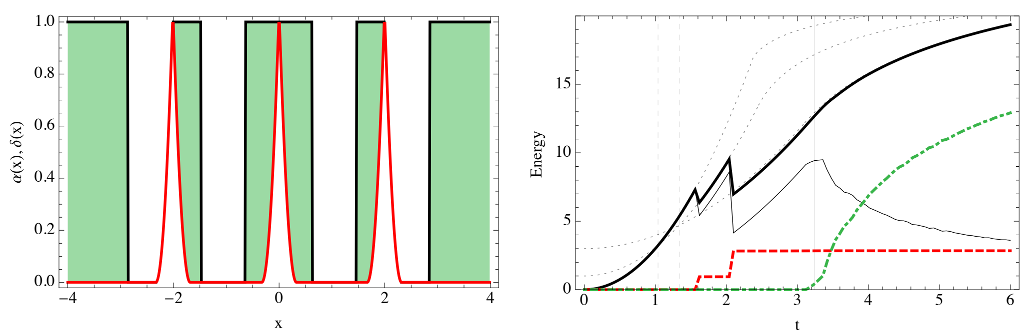

Figure 2. Left: three cracks (red) and debonded regions (green). Right: crack (red), debonding (green) and total (thick black) energies

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