THE INFLUENCE OF SURFACE EFFECTS ON STRENGTH AND STABILITY OF KIRSCH PLATE

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Summary The solution of Kirsch problem in view of surface stress is used to study the influence of surface effects on strength and stability of a planar plate with a circular hole under uniaxial tension. Based on Neuber-Novozhilov's fracture criterion, the destroying force is estimated.

EFFECT OF A SURAFACE STRESS IN KIRSCH PROBLEM

During the last 20-30 years, nanotechnological industry has rather essential achievements. Many devices with nanosize structures exist and work now. In this situation, it is natural the appearance of traditional problems concerning of such structures: strength, fracture, stability, defects, delamination etc. This leads to the necessity of using the special theory to predict and control the behaviour of nanostructures. Such theory was prepared and developed on the base of an idea of surface stresses [1] – [7].

In the presented report, we considered the Kirsch problem on the uniaxial tension of an infinite plate with a circular hole and investigated the influence of surface effects on both the strength problem and stability problem.

The boundary conditions are described by the Laplace-Young law [1], [4]

$$\mathbf{n}\cdot\mathbf{\Sigma}-\nabla_s\cdot\mathbf{\Sigma}^s=0 \tag{1}$$

where $\mathbf{n}$ is the normal unit vector to the boundary, $\mathbf{\Sigma}$ is the tensor of volume stresses, $\mathbf{\Sigma}^s$ is the tensor of surface stresses and

$$\nabla_s\cdot\mathbf{\Sigma}^s=-\left(\frac{s^s_{11}}{R_1}+\frac{s^s_{22}}{R_2}\right)\mathbf{n}+\frac{\mathbf{e}_1}{h_1h_2}\left[\frac{\partial}{\partial a_1}h_2s^s_{11}+\frac{\partial}{\partial a_2}h_1s^s_{21}+\frac{\partial h_1}{\partial a_2}s^s_{12}-\frac{\partial h_2}{\partial a_1}s^s_{22}\right]+$$
$$+\frac{\mathbf{e}_2}{h_1h_2}\left[-\frac{\partial h_1}{\partial a_2}s^s_{11}+\frac{\partial}{\partial a_1}h_2s^s_{12}+\frac{\partial h_2}{\partial a_1}s^s_{21}+\frac{\partial}{\partial a_2}h_1s^s_{22}\right]. \tag{2}$$

Applying equations (1) and (2) to the Kirsch problem with circular hole of radius a and then using Muskhelishvili's technique [8] reduce the problem to the hypersingular integral equation in the unknown function $t(z)=s^s(z)-s^s_0$ [9], where s^s is the surface stress at the boundary of the hole in the loaded plate and s^s_0 is the surface stress at the boundary of the hole in the unloaded plate,

$$\left[2a-M(k-1)\right]t(z)+\frac{M(k+1)}{2pi}\oint_{|h|=1}\frac{\left(h+z^2/h\right)t(h)}{(h-z)^2}dh=Ma(k+1)\left(2c+c_2z^2+\bar{c}_2z^{-2}\right),\qquad |z|=1$$

(3)

As a result, it is possible to get the distribution of the stresses in the polar coordinates r,j near the circular hole

$$s_{rr}(r,j)=\frac{p}{2}\left[1-\frac{a^2}{r^2}+\left(1-\frac{4a^2}{r^2}+\frac{3a^4}{r^4}\right)\cos 2j\right]+d_0\frac{a^2}{r^2}+2d_2\left(\frac{3a^4}{r^4}-\frac{2a^2}{r^2}\right)\cos 2j\ ,$$

$$s_{jj}(r,j)=\frac{p}{2}\left[1+\frac{a^2}{r^2}-\left(1+\frac{3a^4}{r^4}\right)\cos 2j\right]-d_0\frac{a^2}{r^2}+6d_2\frac{a^4}{r^4}\cos 2j\ , \tag{4}$$

$$s_{rj}(r,j)=-\frac{p}{2}\left(1+\frac{2a^2}{r^2}-3\frac{3a^4}{r^4}\right)\sin 2j-2d_2\left(\frac{a^2}{r^2}-\frac{3a^4}{r^4}\right)\sin 2j$$

where

$$d_0=\frac{s^s_0}{a+M}+p\frac{M(1+k)}{4(a+M)},\ d_2=-p\frac{M(1+k)}{2a+M(3+k)},\ M=\frac{2m_s+l_s-g_s}{2m} \tag{5}$$

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In the equations (3)-(5), p is the remote tension, m is the shear modulus of the bulk material, l_s, m_s are the modules of the surface elasticity, similar to the Lamé constants l, m of the bulk material; g_s is the residual surface stress under unstrained conditions.
Formulas (4) coincide with the solution of Tian and Rajapakse [10] obtained by another way.

EFFECT OF A SURAFACE STRESS IN PROBLEM OF STABILITY

Using formulas (4) and the global Timoshenko's idea [11]

$$U = W \tag{6}$$

where

$$U = \frac{D}{2}\left\{ \int_0^{2p}\int_a^{\infty} \left[r\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + \frac{\partial^2 w}{\partial j^2} \right]^2 dr dj \; - \right.$$

$$\left. -2(1-n)\int_0^{2p}\int_a^{\infty} \left[\frac{\partial^2 w}{\partial r^2}\frac{\partial w}{\partial r} - \frac{1}{r}\frac{\partial^2 w}{\partial r^2}\frac{\partial^2 w}{\partial j^2} - \frac{1}{r}\left(\frac{\partial^2 w}{\partial r \partial j}\right)^2 + \frac{2}{r^2}\frac{\partial^2 w}{\partial r \partial j}\frac{\partial w}{\partial j} - \frac{1}{r^3}\left(\frac{\partial w}{\partial j}\right)^2 \right] dr dj \right\},$$

$$W = \frac{h}{2}\int_0^{2p}\int_0^{\infty} \left[s_{rr} r \left(\frac{\partial w}{\partial r}\right)^2 + s_{jj}\frac{1}{r}\left(\frac{\partial w}{\partial j}\right)^2 + 2 s_{rj}\frac{\partial w}{\partial r}\frac{\partial w}{\partial j} \right] dr dj \; ,$$

and works [12] – [14] as well, one can find the external critical force p_{cr} under which the planar plate with the circular hole becomes a deflected one. This way gives us the possibility to study an influence of the surface effect on the process of buckling failure.

Simultaneously, it possible to find destroying external force using the formulas (4) and Neuber-Novozhilov's fracture criterion.

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