THERMAL SHOCK ANALYSIS OF A FUNCTIONALLY GRADED PLATE USING INTERACTION ENERGY INTEGRAL METHOD

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Summary Fractures phenomena can be often found in functionally graded materials (FGMs) subjected to thermal shock loadings. This paper aims to investigate the transient response of a cracked functionally graded plate (FGP) subjected to a thermal shock[1-3]. An analyzing model is proposed by combining the interaction energy integral method and the analytical solutions of the temperature field. Particularly, the transient thermal stress intensity factors (TSIFs) with normalized time and crack length are also investigated.

TRANSIENT THERMAL STERMAL STRESS INTENSITY FACTOR BASED ON A MODIFIED INTERCTION ENERGY INTEGRAL METHOD

Transient temperature field

The unsteady heat conduction equation can be written as

$$\frac{\partial}{\partial x}[\lambda(x)\frac{\partial \tilde{T}(x,t)}{\partial x}] = C(x)\rho(x)\frac{\partial \tilde{T}(x,t)}{\partial t} \tag{1}$$

Based on a perturbation method, the transient temperature field in a FGP (Fig.1) without crack can be expressed as [2]

$$\tilde{T}(\xi,t) = \sum_{m=0}^{\infty} \gamma^m \tilde{T}_m(\xi,t) \tag{2}$$

where γ denotes the small parameter in the perturbation method. $\delta^m \tilde{T}_m(\xi,t)$ can be yielded by applying Laplace transform and residue theorem to the one-dimensional thermal conduction governing equation. It is not necessary to reserve many terms of the series in numerical calculation since the first two terms are proved precise enough. Then, the thermal stress induced by the temperature field can be applied to the finite element model. Afterward, transient TSIFs at crack tips can be computed through a modified interaction energy integral method.

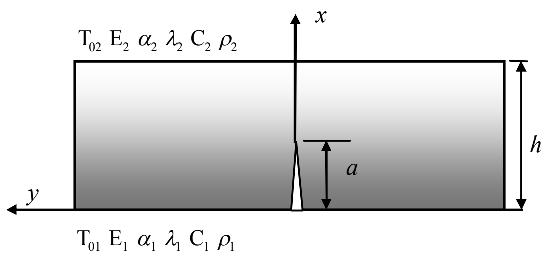

Fig.1. An FGM plate with a surface crack thermal shock

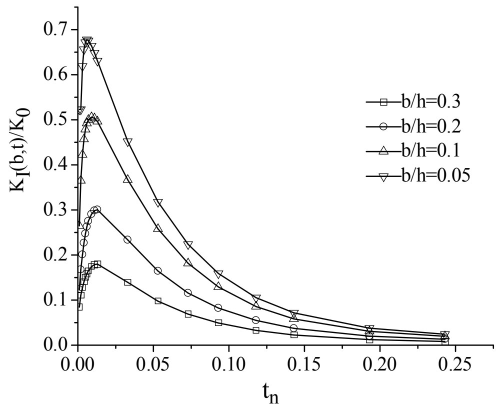

Fig.2. TSIFs for different crack length

Interaction energy integral method for thermal shock

The interaction energy integral can be produced by introducing an auxiliary field ($\sigma^{aux},\varepsilon^{aux},u^{aux}$) in the tradition J-integral. It is actually the interaction term between the actual and auxiliary fields. The corresponding interaction energy integral corresponding to the thermal shock can be obtained based on the method of Guo et al. [3].

$$I = I_{mechaical} + I_{temperature} \tag{3}$$

The total strain includes the mechanical strain ε^e and thermal strain ε^t. Substituting the mechanical strain ε^e into the expression of the interaction energy integral, we can obtain

$$I_{mechaical} = \int_A \left(\sigma_{ij}^{aux} u_{i,1} + \sigma_{ij} u_{i,1}^{aux} - \sigma_{ij}^{aux} \varepsilon_{ik} \delta_{1j}\right) q_{,j} dA + \int_A \sigma_{ij} \left[S_{ijkl}^{tip} - S_{ijkl}(x)\right] \sigma_{kl}^{aux} q dA \tag{4}$$

$$I_{temperature} = \int_A \sigma_{ij}^{aux} \alpha_{,1} \left(T - T_0\right) q dA \tag{5}$$

The mode-I stress intensity factor can be expressed as

$$K_1 = (E'_{tip}/2) I \tag{6}$$

Thus, the model-I transinent TSIFs can be obtained from the interaction energy integral.

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NUMERICAL EXAMPLES AND RESULTS ANALYSES

Distribution of thermomechanical properties of FGMs

A ceramic/metal FGM plate with a surface crack is considered. An infinitely long FGP with its thermomechanical properties varying along the thick direction is shown in Table1. All the properties are defined by exponential functions [4].

Table 1. Thermophysical properties of the model materials [4]

materials	Specific heat (J/kgK)	Thermal conduc-tivity (W/(Mk))	Coefficient Of thermal expansion ($\times 10^{-6}$/K)	Density (kg/m^3)	Young' Modulus (GPa)	Poisson's ratio
ceramic	$c_1 = 456.7$	$k_1 = 2.09$	$\alpha_1 = 10.0$	$\rho_1 = 5,331$	$E_1 = 151.0$	$\nu_1 = 0.33$
metal	$c_2 = 537.0$	$k_2 = 7.5$	$\alpha_2 = 9.5$	$\rho_2 = 4,420$	$E_2 = 116.7$	$\nu_2 = 0.33$

SIFs in FGP under transient thermal shock

To verify the present method, a representative numerical example is considered. As shown in Fig.1, the thickness h and the normalized crack length b/h are chosen as 0.05, 0.1, 0.2 and 0.3, respectively. The initial temperature is $T_0 = 1000K$. The sudden temperature changes (T_{01}=−700 at $x=0$ and T_{02}=−600 at $x = h$) imposed on both surfaces of the plate are considered. It should be noted that the heat will flow only along the x-direction. The normalized factor and the normalized time are defined as $K_1 = \sqrt{\pi a}\alpha_1 T_{01} E_1/(1-\nu)$ and $t_n = (\lambda_1 / c_1\rho h^2)t$, respectively. Table2 depicts comparison between the present results and analytical results given by Noda and Guo[4]. Excellent agreement can be observed.

Table 2. present results and analytical results given by Noda and Guo[4]

	TSIFs peak value				TSIFs sletady value			
Ref[4]	$t_n = 0.012$	0.503	$t_n = 0.012$	0.178	$t_n = 0.193$	0.030	$t_n = 0.193$	0.012
Present	$b/h = 0.3$	0.4974	$b/h = 0.1$	0.1761	$b/h = 0.3$	0.0283	$b/h = 0.1$	0.0113

CONCLUSIONS

Fractures phenomena can be often found in functionally graded materials (FGMs) subjected to thermal shock loadings. Due to the nonhomogeneity of FGMs, interaction energy integral methods have been developed to obtain the stress intensity factors. However, these methods are mainly applied in static or steady crack problems. Firstly, this paper a perturbation method is adopted to obtain the transient temperature field corresponding to the thermal shock. Then, the temperature field is applied to the finite element model to evaluate the displacements, stresses and strains. Particularly, in order to obtain the transient thermal stress intensity factors (TSIFs), a modified interaction energy integral method is adopted. The variation of the transient TSIFs with normalized time and crack length are also investigatedin Fig.2. The results revealed the larger hazard for FGMs under thermal shock loadings than under steady conditions. Moreover, the model is proved to be effective and efficient for the transient thermal fracture analysis of the FGP.

Acknowledges

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