ANALYSIS OF THE INFLUENCE OF STRESS GRADIENT ON THE NON-PROPAGATING FATIGUE CRACK

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Summary The stress gradients near the elongated notch root along with the propagation of short crack and the resulting non-propagating crack lengths a_{npc} have been estimated. The a_{npc} can be estimated from known material properties, specimen's geometry as well as applied loading. The characteristic size of short crack a_0 associates with the fatigue stress concentration factor K_f. The estimated values of a_{npc} are in fairly good agreement with available experimental values.

INTRODUCTION

The non-propagating fatigue cracks have been reported during the past decades and many theories have been employed to justify the observations on non-propagating cracks [1]. In some cases especially for the shallow notches, cracks may initiate under cyclic loading and then stop or become non-propagating because of the significant stress gradient. In this paper, a simple method is proposed to estimate the behavior of non-propagating crack. Our innovation is that an improved model is proposed to estimate the behavior of non-propagating crack based on the originality that the fatigue stress concentration value reduces from the static stress concentration factor K_t to the fatigue stress concentration factor K_f along with the propagation of short cracks. The proposed approach can be utilized in developing a simple model for the quantification of a_{npc} and K_f. The derived nominal stress and the non-propagating crack size have been experimentally verified by several specimens of different materials in the literatures.

Stress gradient near the sharp notch

The stress concentration is usually caused by geometrical discontinuity or heterogeneity of microstructure which involves an appearance of the maximum stress compared to the calculated median values based on the smooth section. We place ourselves here in the investigation of the plate containing a crack length located at the bottom of an elliptic notch, the stress σ_y at the point ($x \geq b$, 0), which acts in the remaining ligament of an infinite plate with an elliptic hole, is given by the Eq. (1) [2]:

$$\frac{\sigma_y}{\sigma_n}-1=\frac{\left(b^2-2bc\right)\left(x-\sqrt{x^2-b^2+c^2}\right)\left(x^2-b^2+c^2\right)+bc^2\left(b-c\right)x}{\left(b-c\right)^2\left(x^2-b^2+c^2\right)\sqrt{x^2-b^2+c^2}} \tag{1}$$

The σ_y/σ_n can be defined as the stress concentration factor in the presence of short crack K_{tf}. Thus, the gradient of σ_y at the bottom of notch (at the edge of the elliptic hole) is given by the following equation:

$$d\sigma_y/dx\big|_{x=b}=-\left(2K_t+1\right)\sigma_n/\rho=-\left(3+4b/c\right)b\sigma_n/c^2 \tag{2}$$

The relationship between the stress concentration K_t (without crack) and the stress concentration ($K_{tf} = \sigma_y/\sigma_n$) (in the presence of a crack) is practically linear and not very sensitive to the value of K_t. The ratio between K_{tf} and K_t falls steeply from 1 for a point located at the end of the notch to 0.9 for a point which is very close to the distance of $x = b/20$. This rather high stress gradient can cause an initiation of fatigue short crack, even in the case of a nominal stress lower than the yield stress. In next section, we will discuss the behavior of the short crack.

Discussions and comparisons

The value of ΔK used to describe the propagation of the fatigue cracks have been developed by Meggiolaro [3] as described in Eq. (3). The factor $\varphi(a)$ includes the stress concentration effect, which can be regarded as K_{tf}, and α encompasses the remaining terms, such as the free surface correction.

$$\Delta K=\alpha\cdot\varphi(a)\cdot\Delta\sigma_n\sqrt{\pi(a+a_0)} \qquad \text{with} \qquad a_0=\frac{1}{\pi}\left(\frac{\Delta K_0}{\alpha\cdot\Delta S_0}\right)^2 \tag{3}$$

The redistribution of the stress near the crack tip along with the propagation of short crack can be obtained by Eq. (4).

$$\Delta\sigma(a) = \varphi(a)\cdot\Delta\sigma_n \tag{4}$$

The formulas of the $\varphi(a)$ can be properly obtained in the literatures [4]. Compared to the Kitagawa diagram, the crack propagation criterion based on the functions $f(a, \rho, \Delta\sigma_n, \Delta S_0, \Delta K_0)$ is proposed in Eq. (5).

$$\begin{cases} f=\varphi(a)\cdot\Delta\sigma_n-\Delta S_0>0 & \text{for } a<a_0 \\ f=\varphi(a)\cdot\Delta\sigma_n-\dfrac{\Delta K_0}{\alpha\sqrt{\pi a}}>0 & \text{for } a\geq a_0 \end{cases} \tag{5}$$

By the definition, K_f can be found from the equation:

$$K_f=\frac{\Delta S_0}{\Delta\varphi_n}=\varphi(a_0) \tag{6}$$

Non-propagating crack lengths have been measured by several workers. Data were taken from Frost and Dugdale[1] and Tanaka and Nakai[5] and compared with the predictions derived above as shown in Table 2.

Material	ρ (mm)	$\Delta\sigma_0$ (MPa)	ΔK_0 (MPa√m)	Notch depth D(mm)	$\Delta\sigma_n$ (MPa)	a_{npc0} (mm)			a_{npc} (mm)	a_0	K_f	K_t
Mild steel [5]	0.25	400	12	10.16	42	0.1	0.2		>0.12	0.23	6.72	13.75
Mild steel [5]	0.25	400	12	10.16	57.8	0.4	0.5	0.6	>0.21 <0.85	0.23	6.72	13.75
Mild steel [5]	0.1	400	12	15	30.9	0.1			>0.1	0.23	8.14	25.5
Mild steel [5]	0.051	400	12	20.32	27.8	0.5	0.6	1.1	>0.1	0.23	9.45	40.9
Mild steel [5]	0.051	400	12	20.32	30.6	0.3	0.4		>0.12	0.23	9.45	40.9
Mild steel [5]	0.051	400	12	20.32	38.9	0.5	0.6		>0.2	0.23	9.45	40.9
steel [20]	0.16	326	12.22	6	60.7	0.099			>0.018 <0.15	0.36	2.24	13.2
steel [20]	0.16	326	12.22	6	69.3	0.28			>0.019 <0.15	0.36	2.24	13.2
steel [20]	0.16	326	12.22	6	86.7	0.5			>0.05 <0.14	0.36	2.24	13.2
steel [20]	0.16	326	12.22	6	88.4	1.41			>0.05 <0.14	0.36	2.24	13.2
steel [20]	0.16	274	8.36	6	52	0.039			>0.018 <1.5	0.24	2.0	13.2
steel [20]	0.16	274	8.36	6	60.7	0.25			>0.017 <0.63	0.24	2.0	13.2
steel [20]	0.16	244	6.38	6	43.3	0.029			>0.02 <0.8	0.17	1.94	13.2
steel [20]	0.16	244	6.38	6	52	0.125			>0.03 <1.4	0.17	1.94	13.2
steel [20]	0.16	244	6.38	6	60	0.37			>0.1 <0.7	0.17	1.94	13.2

Table 1: Comparisons of predicted and tested data for two materials

The experimental results are in good agreement with the model for the two materials. In the present model the minimum non-propagating crack length a_{npc} is dependent upon the $\Delta\sigma_n$, D, ΔS_0 and ΔK_0. The K_f depends upon not only the D, ρ, but also the $\Delta\sigma_0$, ΔK_0. We can find that the stress concentration defined by the $\varphi(a) = \sigma_y/\sigma_n$ decreases with the propagation of short crack near the notch root and the minimum $\varphi(a)$ value which could produce a non-propagating crack occurs when the short crack length arrives a_0. According to the definition of K_f, the K_f value can be substituted by $\varphi(a_0)$. The comparisons between the K_f and K_t (by Inglish method) have also been demonstrated in Table 1.

CONCLUSIONS

As discussed in this paper, fatigue short crack growth from an elongated notch is highly influenced by the stress gradient near the notch root. A crack which initiates from a sharp notch root due to the elevated stresses may stop growing once the maximum stress near the crack tip drops below the Kitagawa curve. This is because the stress concentration decreases resulting from the redistribution of the stress along with the short crack growth. Correspondingly, a simple method has been proposed for the estimation of the K_f, which is proposed to be the decreased stress concentration factor caused by the short crack propagation. By using this method it has been possible to accurately estimate the non-propagating crack length. The predicted values fall in the range of the experimental results of the literatures. The method proposed here can be applied to explain the difference between the K_t and K_f and further to precisely estimate the K_f and a_{npc}.

References

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